

$$\frac{1}{X+i} \cdot \frac{1}{X+j} = \frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$$

$$\mathcal{C} = \begin{bmatrix} \frac{1}{a_1 - b_1} & \frac{1}{a_1 - b_2} & \dots & \frac{1}{a_1 - b_n} \\ \frac{1}{a_2 - b_1} & \frac{1}{a_2 - b_2} & \dots & \frac{1}{a_2 - b_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_m - b_1} & \frac{1}{a_m - b_2} & \dots & \frac{1}{a_m - b_n} \end{bmatrix}$$

Batch Encryption Using “Partial Fractions”

Using rational functions to do cool crypto (part 2)
(See EUROCRYPT 2026 for part 1!)

$$\sum \frac{1}{X+k}$$

Rohit Nema

joint work with



Dan Boneh
(Stanford)



Arnab Roy
(Mysten Labs)



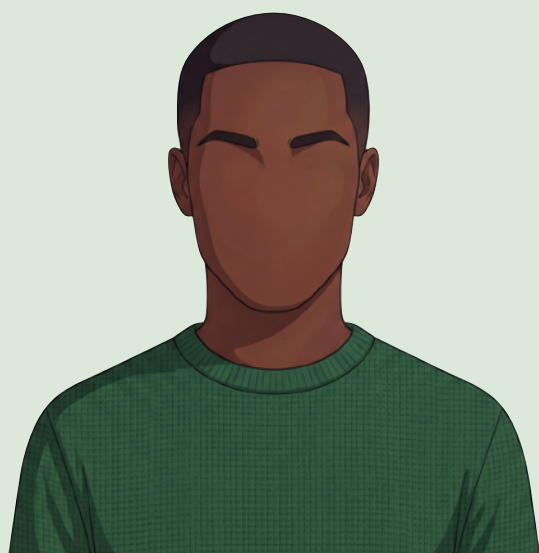
Ertem Nusret Tas
(a16z Crypto Research)

Paper: ia.cr/2026/674

$$\frac{1}{(X+k)^2}$$

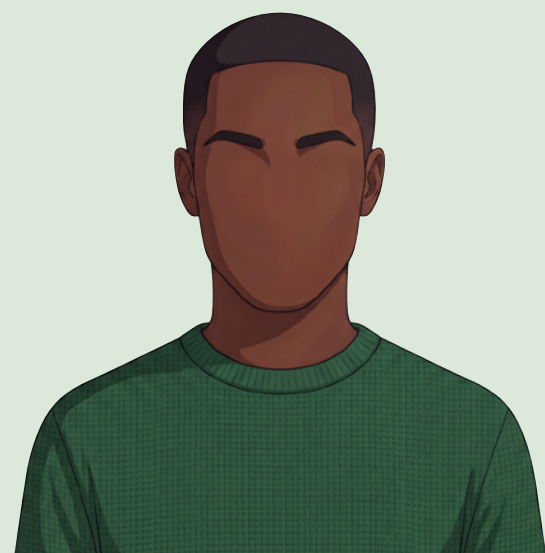
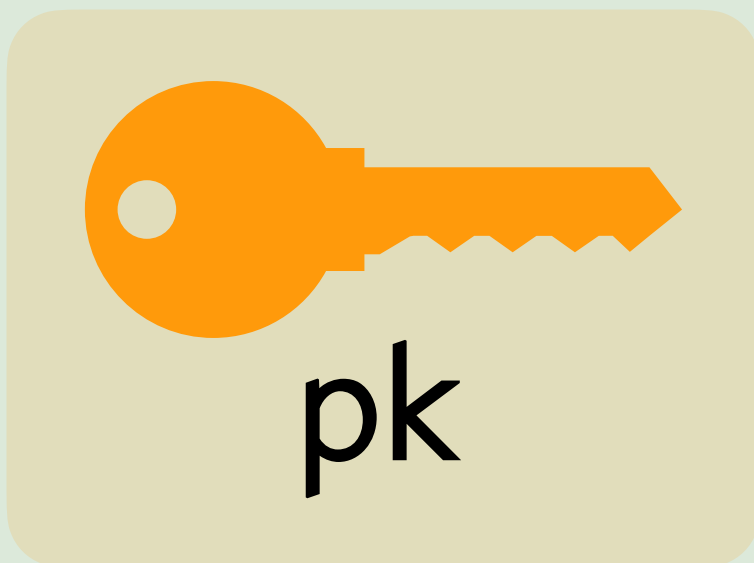
Batch Encryption

introduced by [CGPP24]



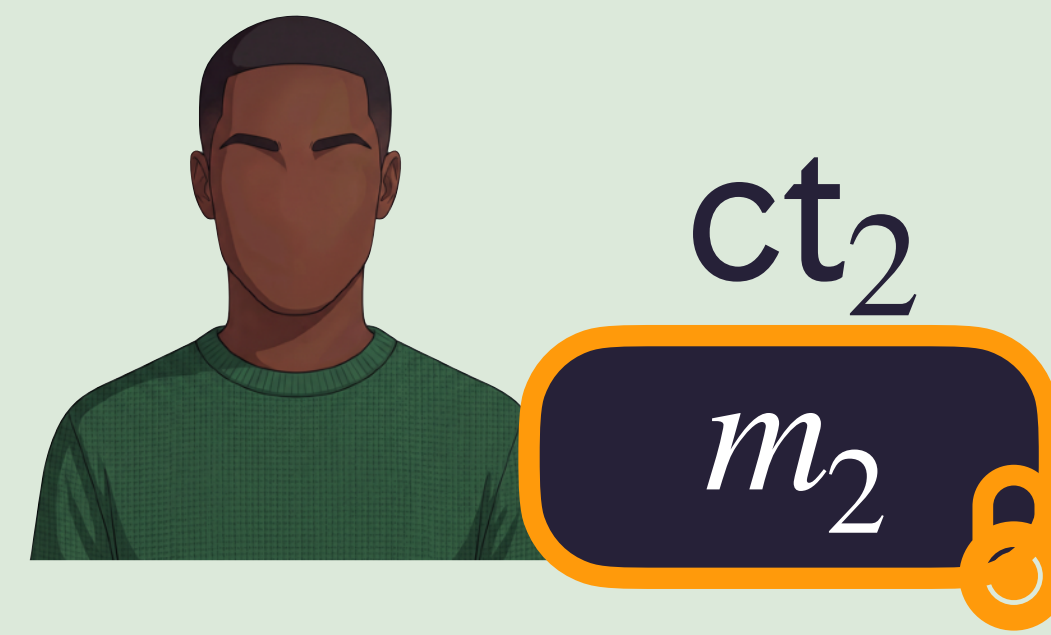
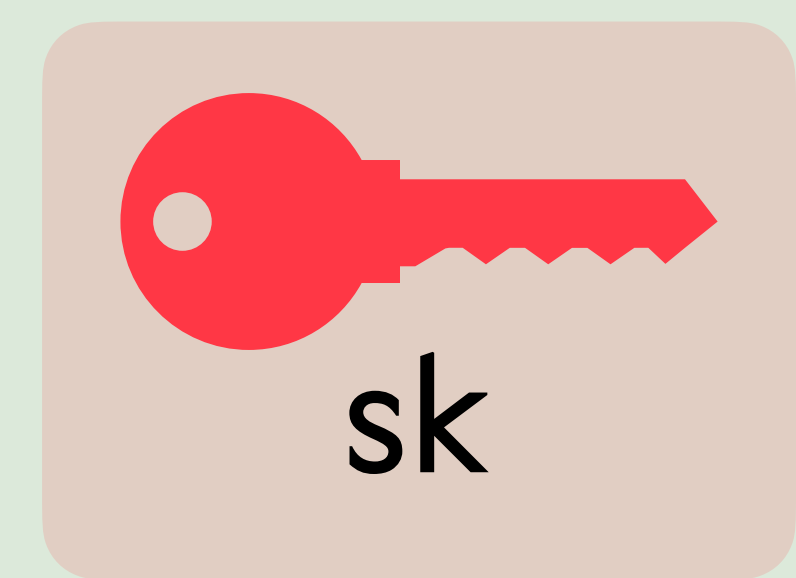
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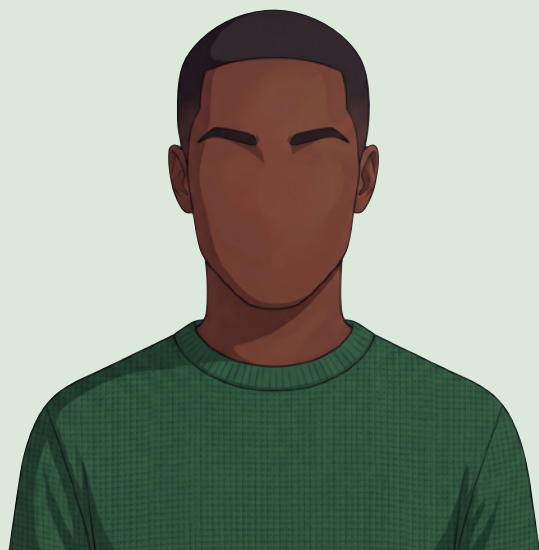
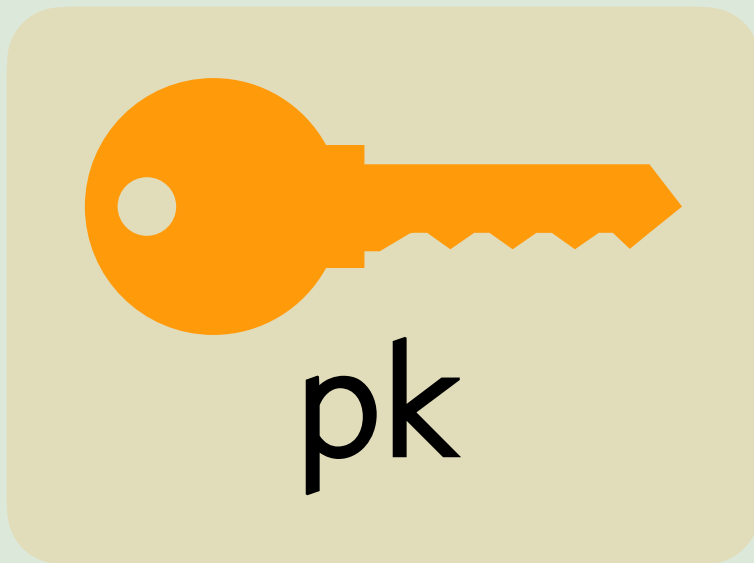
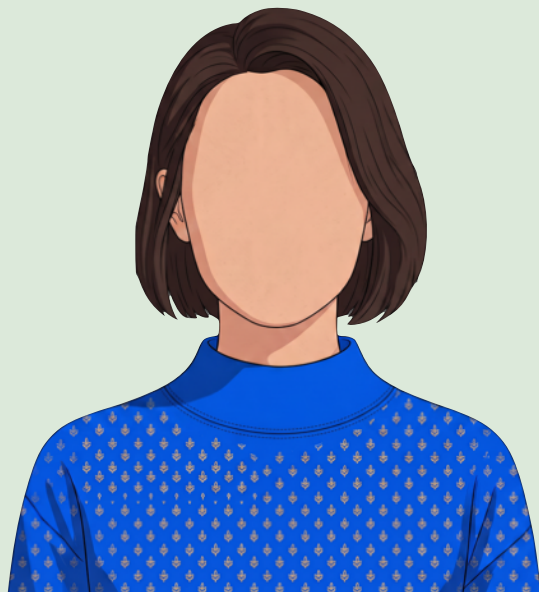
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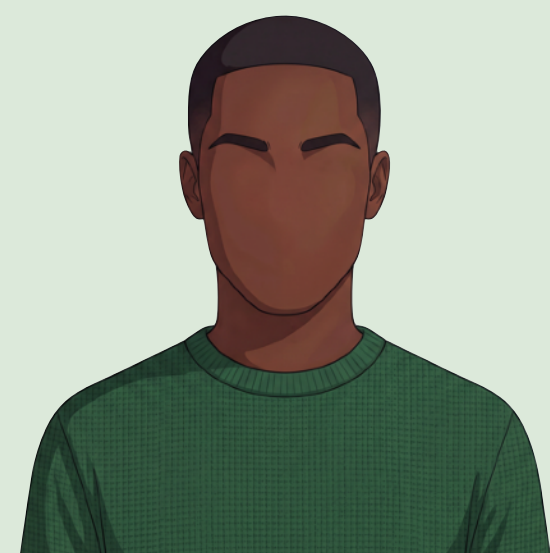
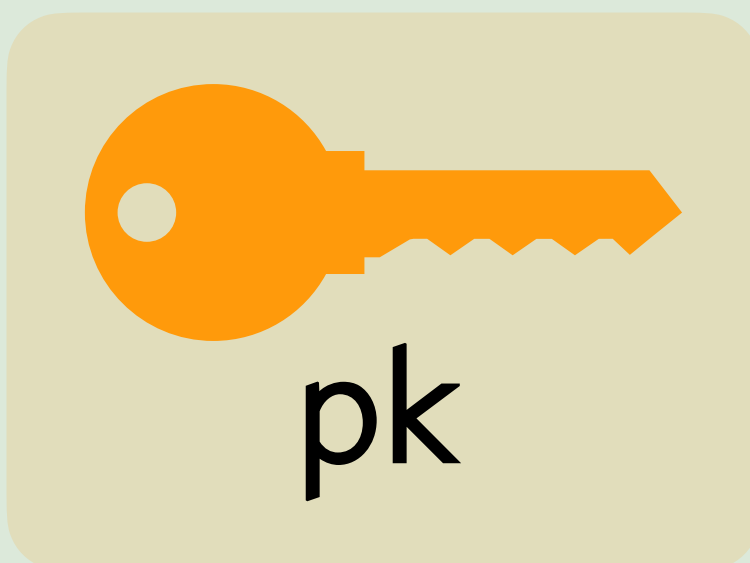
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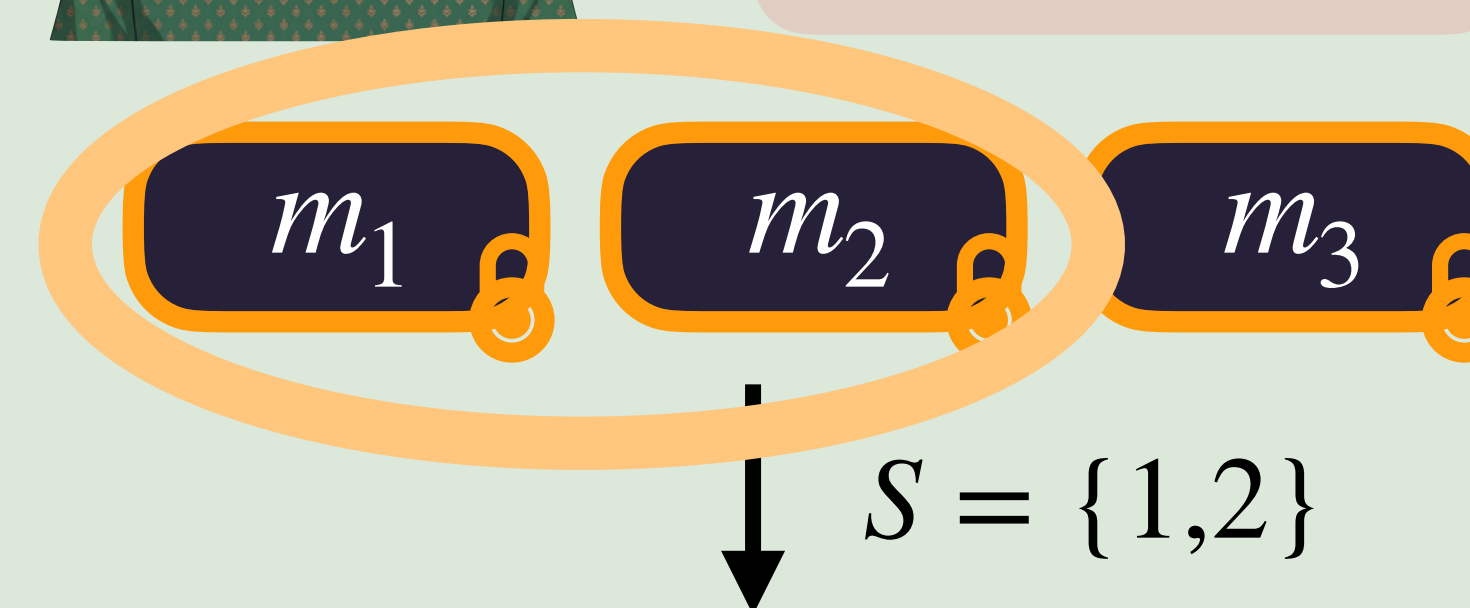
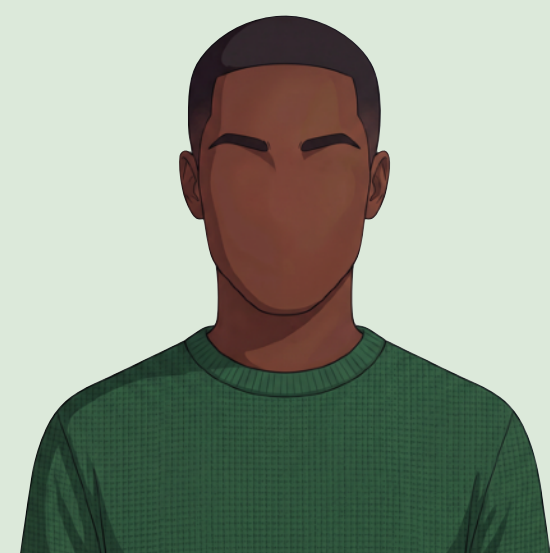
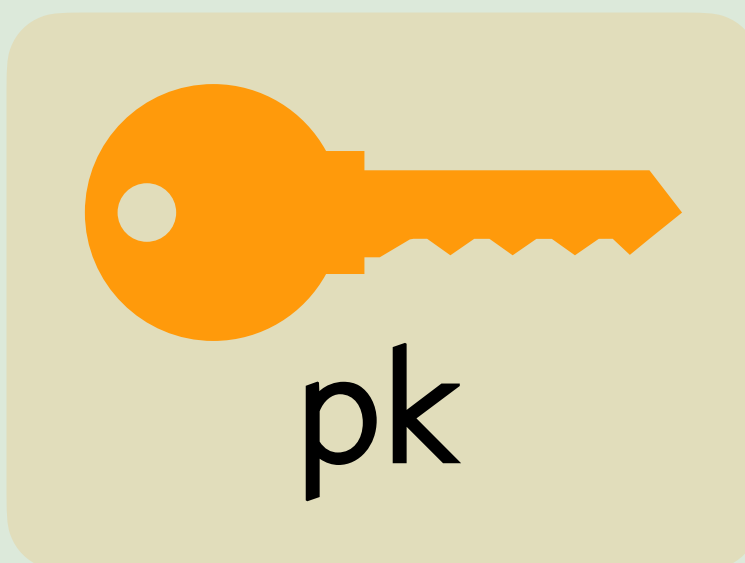
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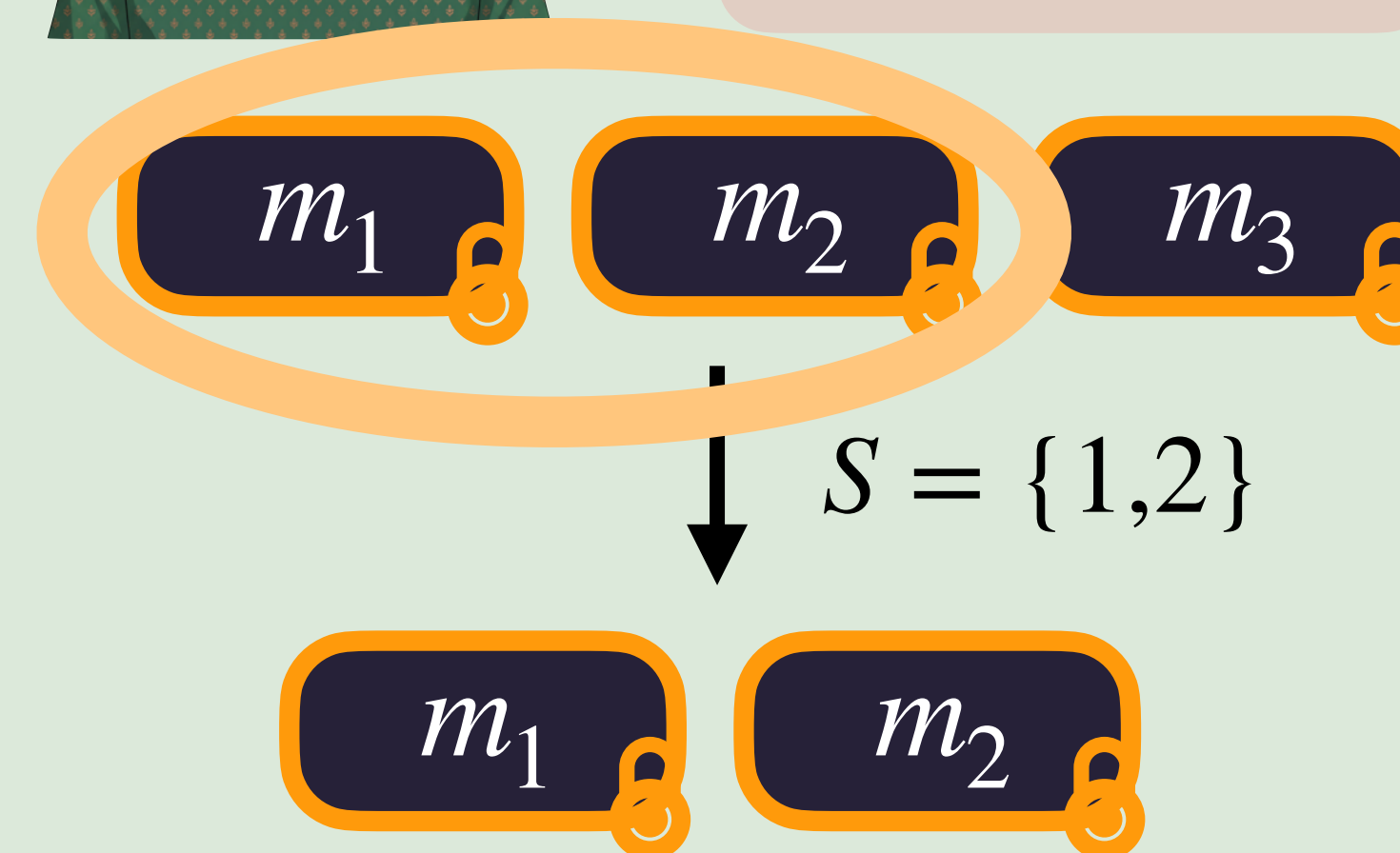
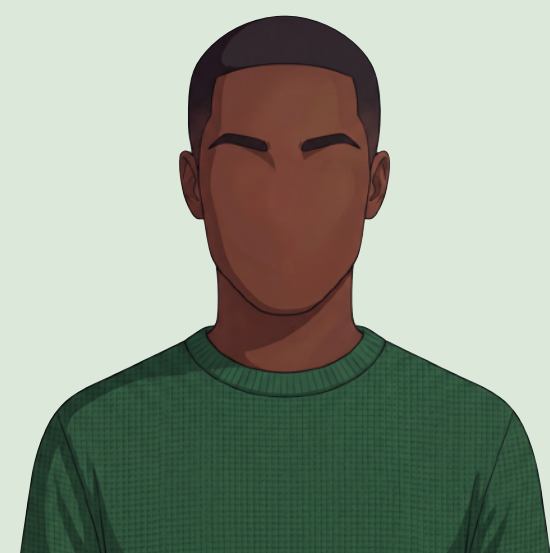
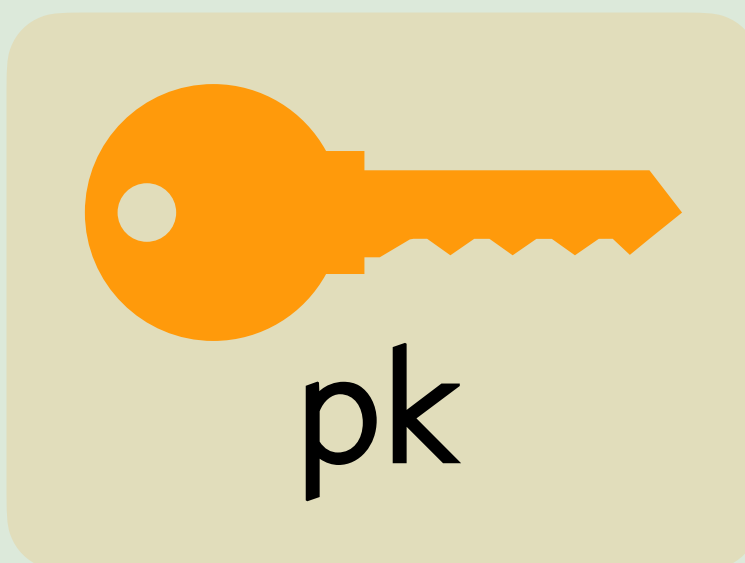
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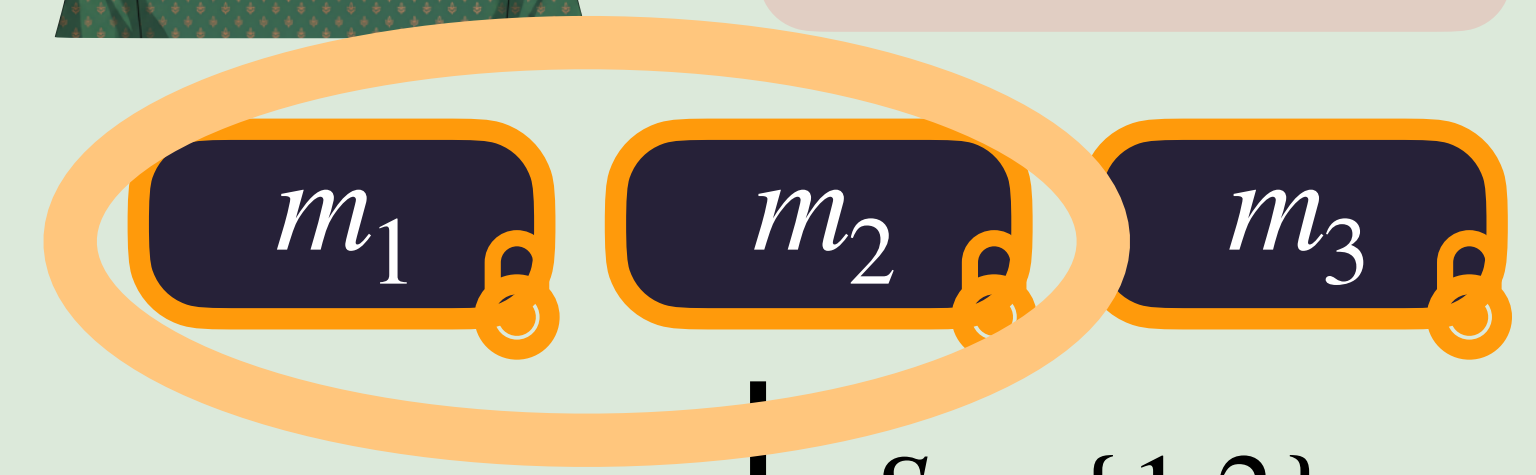
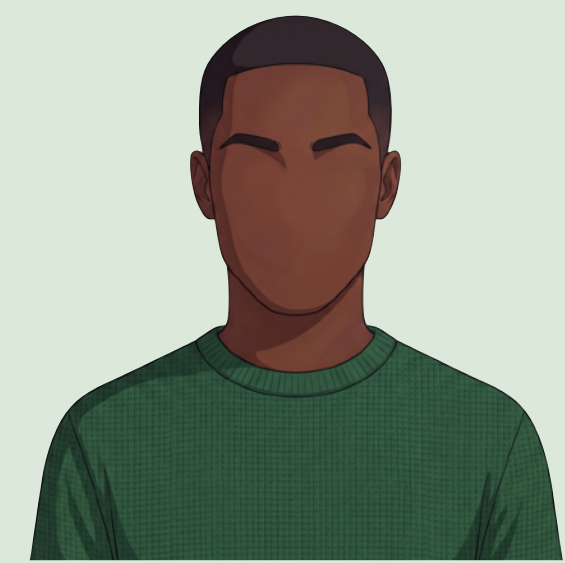
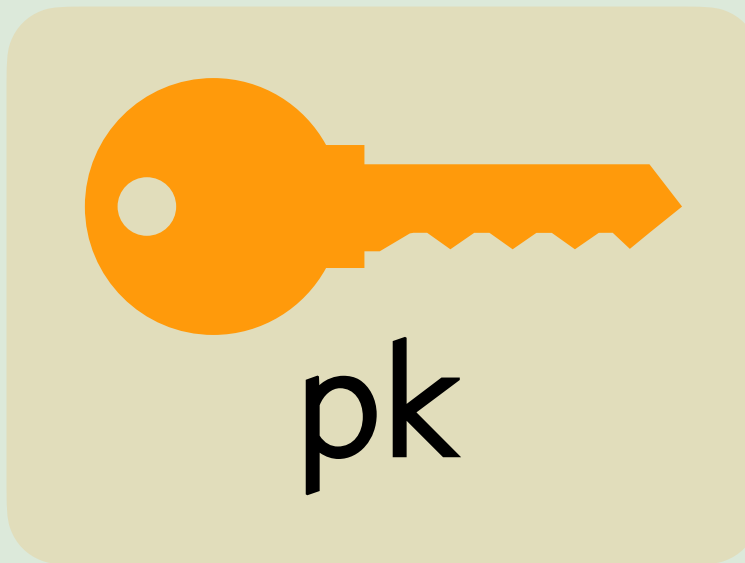
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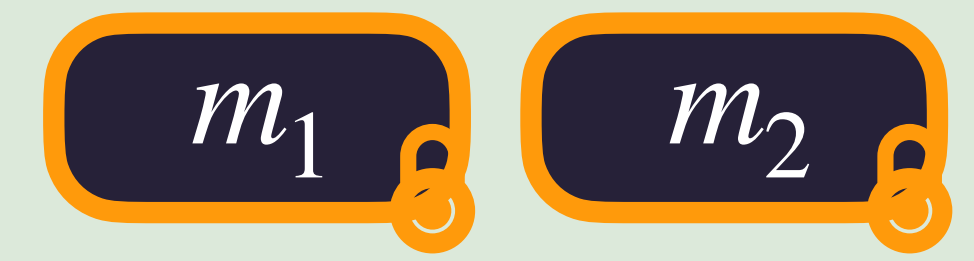


Batch Encryption

introduced by [CGPP24]



$S = \{1, 2\}$



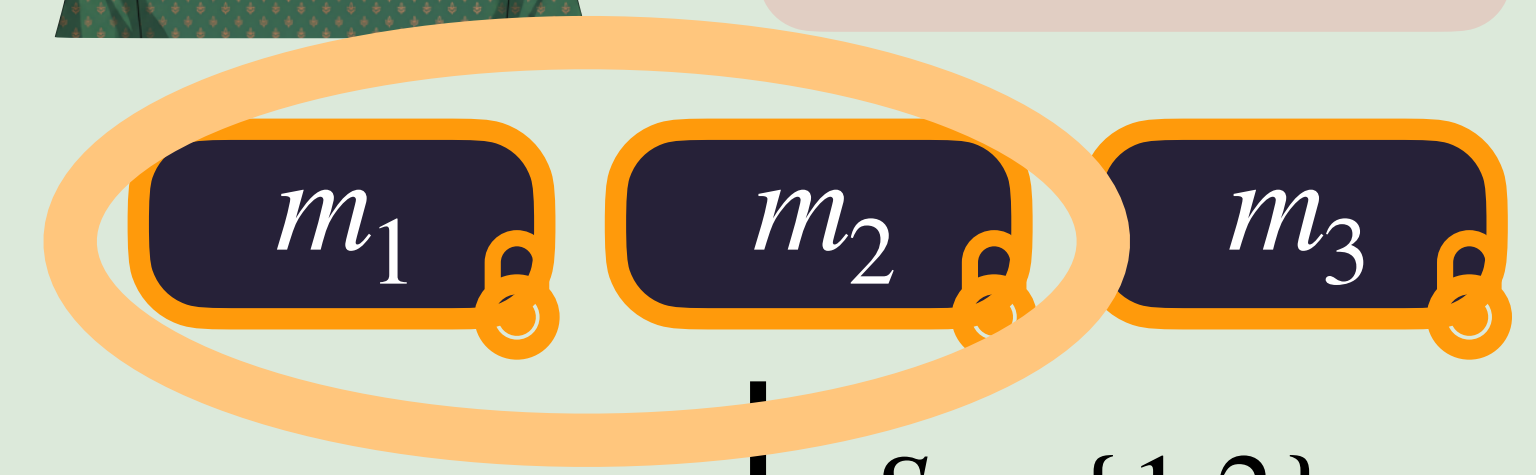
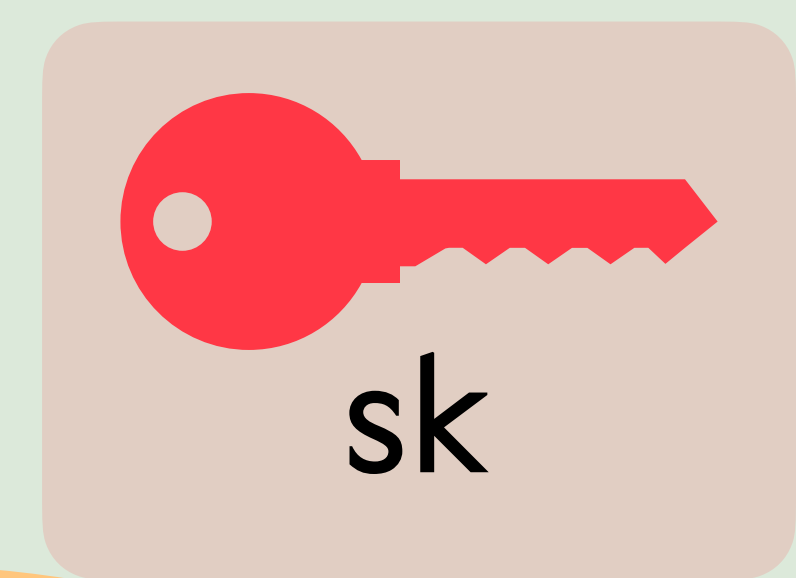
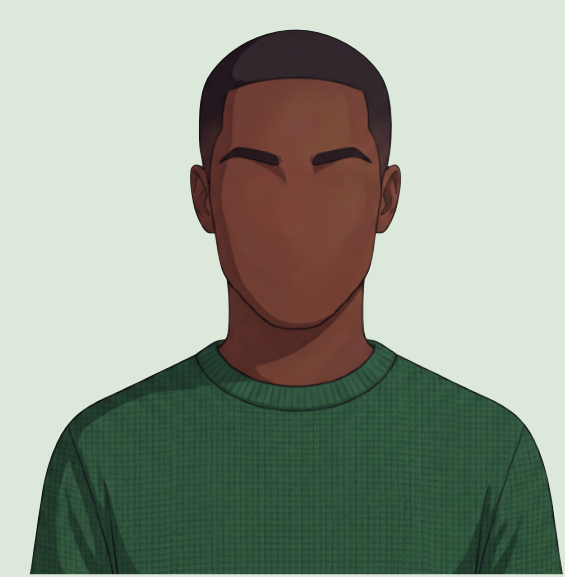
derive
“pre-decryption”
key



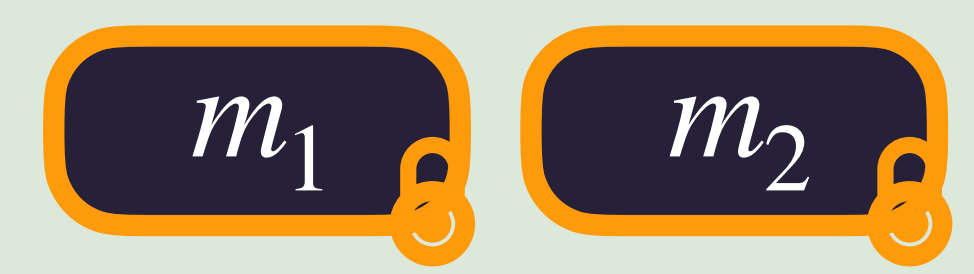
sbk

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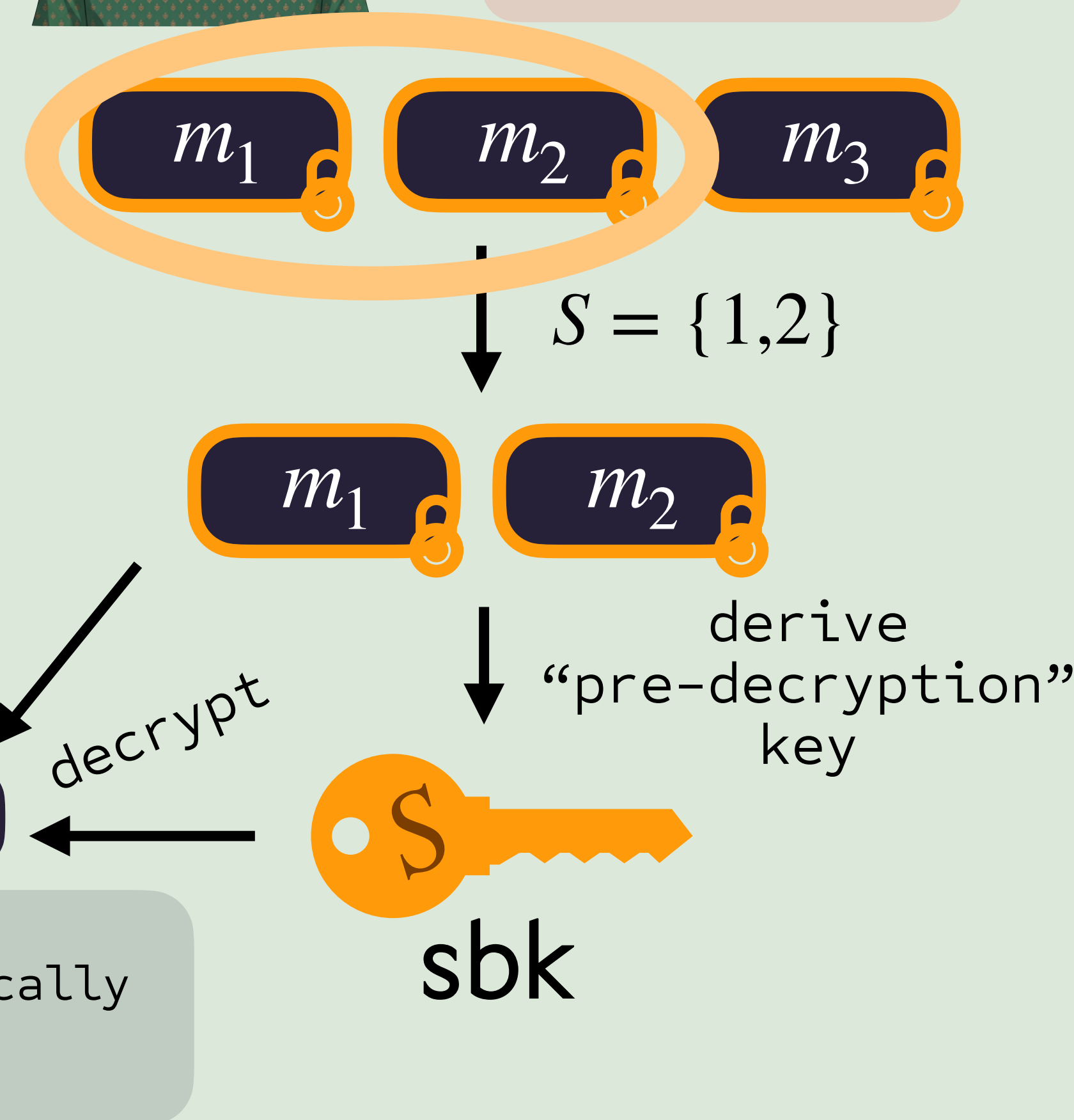
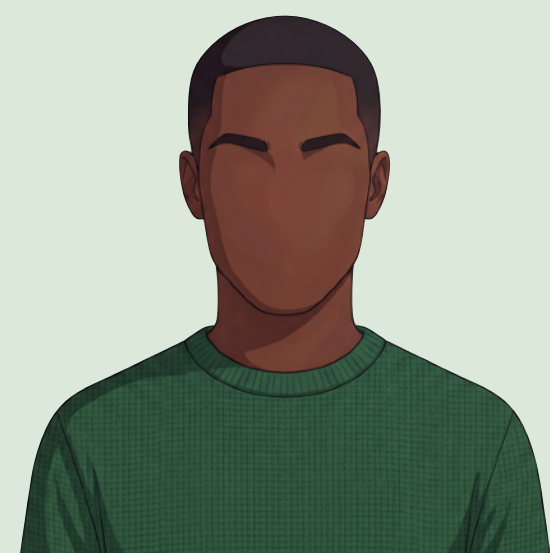
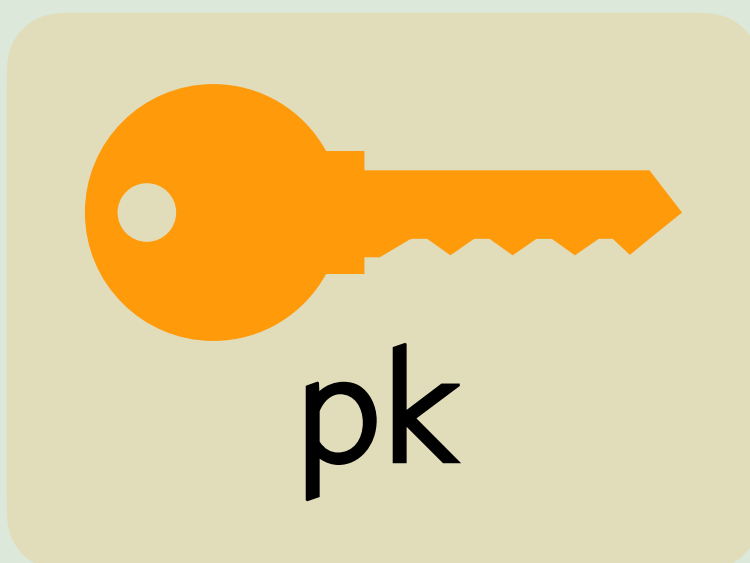
decrypt



sbk

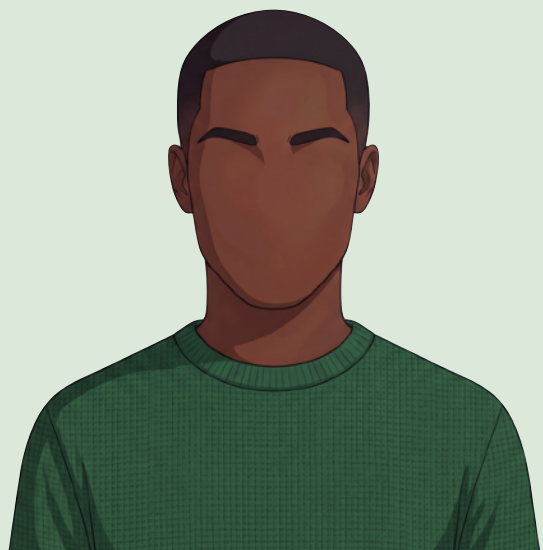
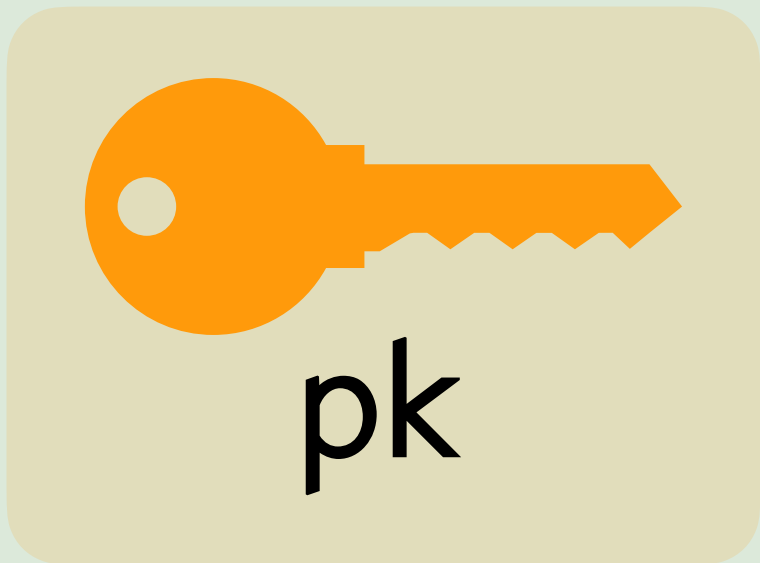
Batch Encryption

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Batch Encryption

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m_1

m_2

m_3

trivial solution:
Decryptor can simply do the decryption themselves and publish the decrypted messages as sbk

m_1

m_2

decrypt

live
pre-decryption"
key

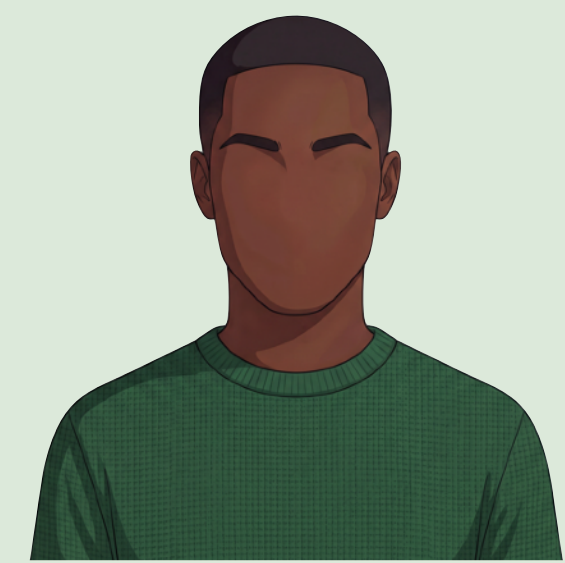


m_3

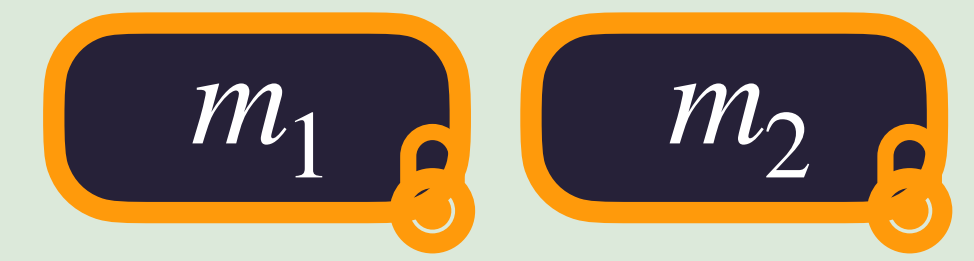
remains semantically secure

Batch Encryption

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derive
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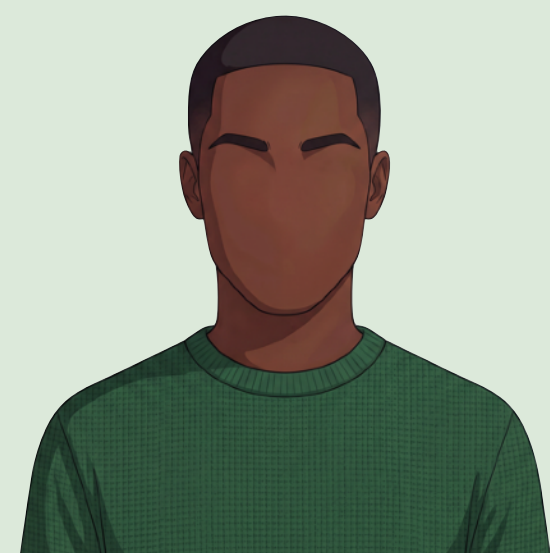
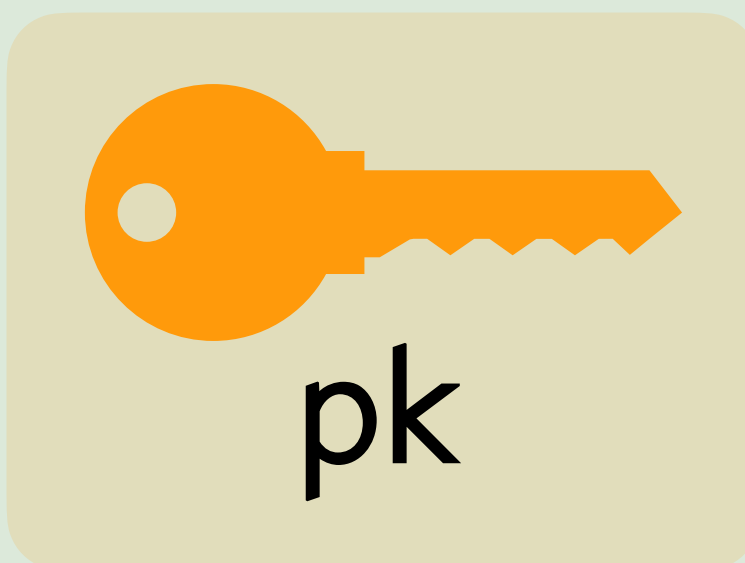
decrypt

pre-decryption
key should be
short



Batch Encryption

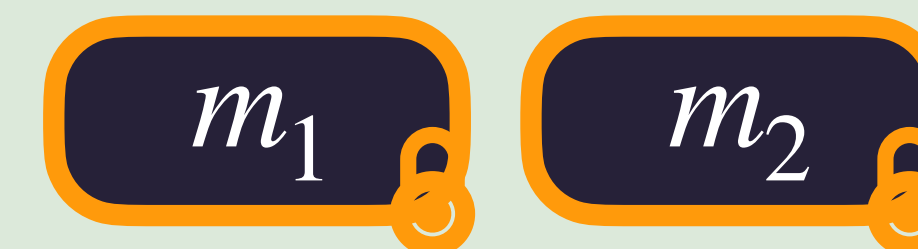
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ciphertexts
should also be
short



$S = \{1, 2\}$



derive
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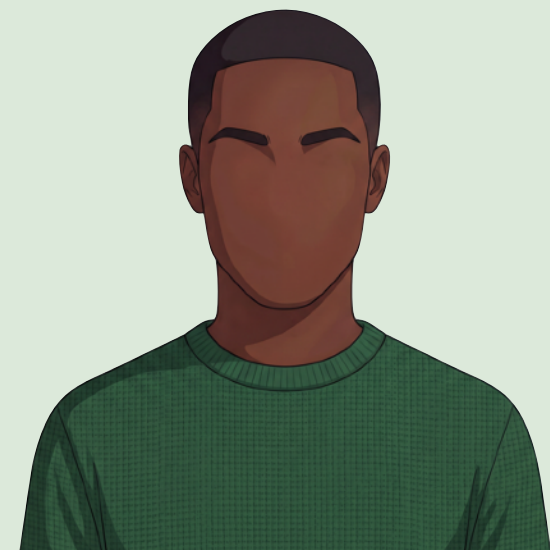
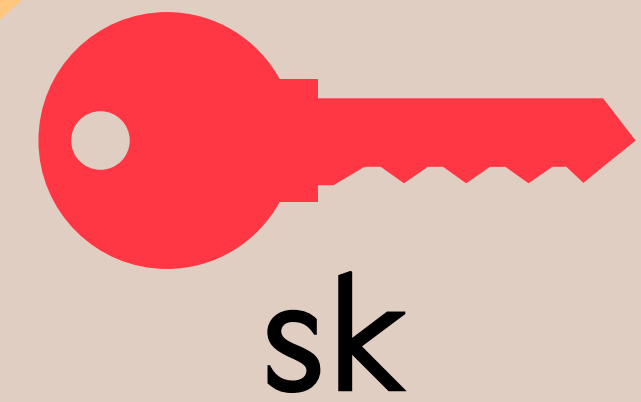
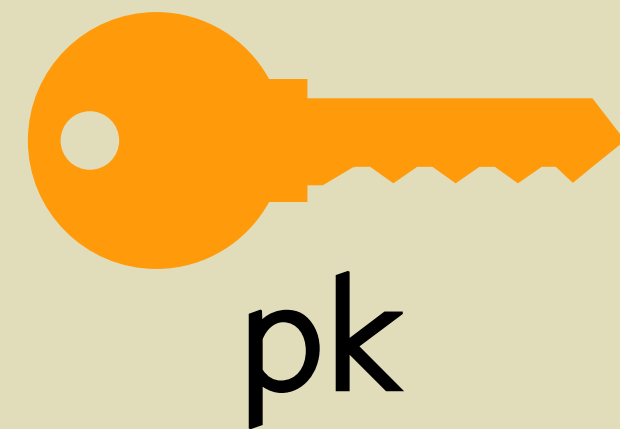
decrypt

pre-decryption
key should be
short



Batch *Threshold* Encryption

introduced by [CGPP24]



can be split between multiple parties with



and threshold t , s.t. at least t parties required to decrypt subset.

Decryption is non-interactive!

m_1

m_3

remains semantic secure

Batch *Threshold* Encryption

Batch *Threshold* Encryption

Highly active area of research

Loads of work on this in various settings!

See e.g. [SAS24, ABD+25, AFP25, BCF+25, BF0+25, CGP+25, FPT+25, GWW+25, ADG+26, Pol26]

Achieve either a **relaxed notion of the primitive** or **suffer from a large CRS**. (We will revisit this toward the end...)

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Achieve either a **relaxed notion of the primitive** or **suffer from a large CRS**. (We will revisit this toward the end...)

Why do we care?

Batch encryption has a **strong application in blockchains**.

It **allows transactions to stay hidden (encrypted)** until the block is confirmed.

Helps **prevent MEV attacks** (e.g. front-running)

The Core Technique

[Jutla-N-Roy26]

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Use *simple* rational functions as first-class primitives

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$$\frac{1}{X + c}, c \in \mathbb{F}_p$$

finite field of prime order p

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These fractions have two elementary yet powerful properties:

(1) Decomposition

(2) Linear Independence

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we'll focus on
this one for this
talk

(1) Decomposition

(2) Linear Independence

Property 1: Decomposition

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Property 1: Decomposition

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{X+i} \cdot \frac{1}{X+i}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{X+j} \cdot \frac{1}{X+j}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{X+k} \cdot \frac{1}{X+k}$

Matrix of pairwise products

Property 1: Decomposition

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{(X+j)^2}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{(X+k)^2}$

Products on the *diagonal* do not decompose. Remain *quadratic*.

Property 1: Decomposition

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$
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$-k$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$	$\frac{1}{k-j} \left(\frac{1}{X+k} - \frac{1}{X+j} \right)$	$\frac{1}{(X+k)^2}$

Off-diagonal/cross-terms *linearize!*

Our Construction: Roadmap

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Index-dependent batch encryption

Ciphertext is tied to a
fixed position in the
batch **known at encryption**
time

$\text{Enc}(\text{pk}, \text{index } i \in [\ell], \text{msg } m)$

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Broadcast Encryption

Short ciphertext can be
decrypted by only
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$\text{Dec}(\text{authorized } \text{sk}_j, \text{ct}) \rightarrow m$
 $\text{Dec}(\text{unauthorized } \text{sk}_j, \text{ct}) \rightarrow \perp$

from [JNR26]:
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Batch Encryption

Thresholdize
using LSS



Batch Threshold Encryption

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today's focus:
we will construct this using the *partial fractions* technique we saw earlier from [JNR26]:
built using partial fractions

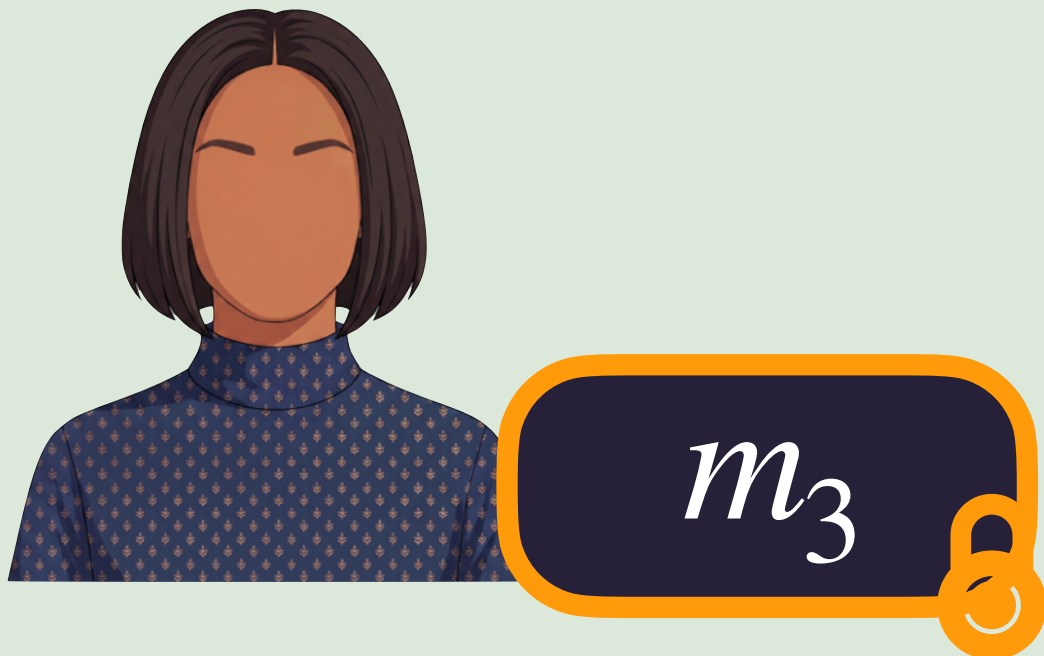
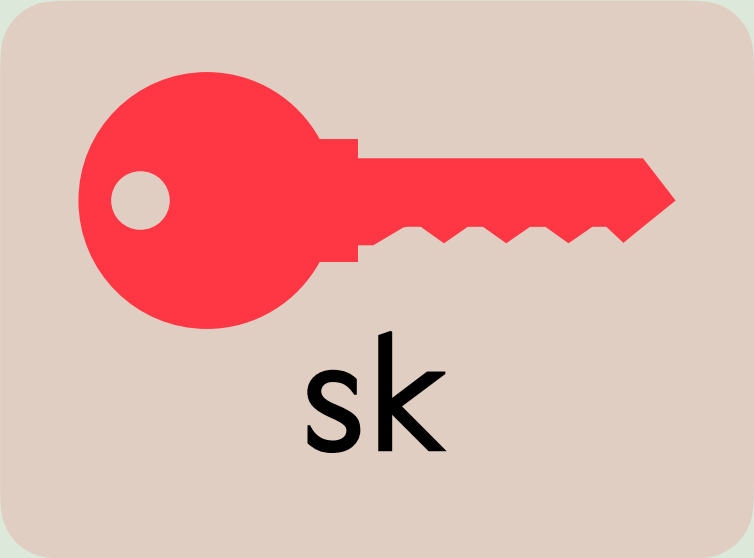
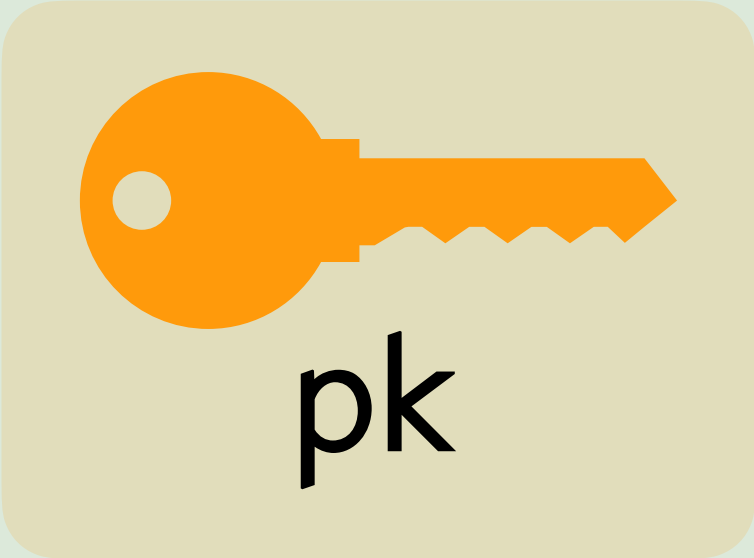
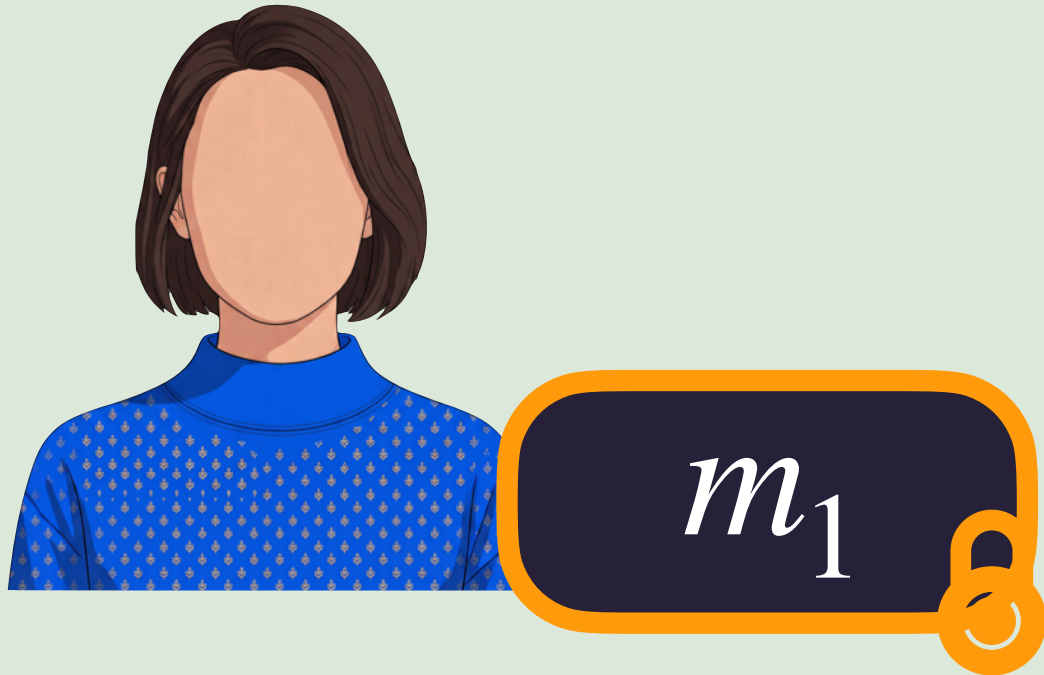
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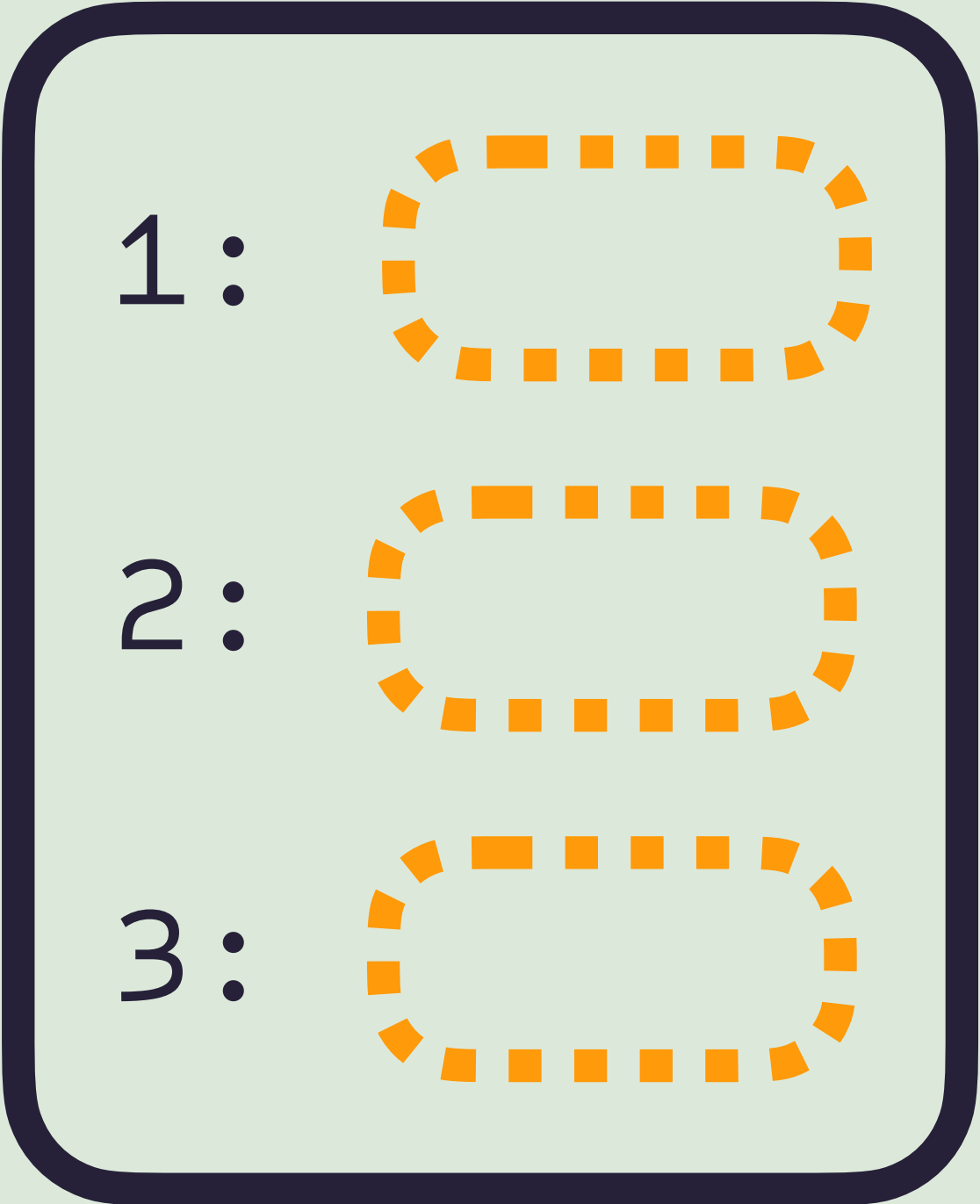
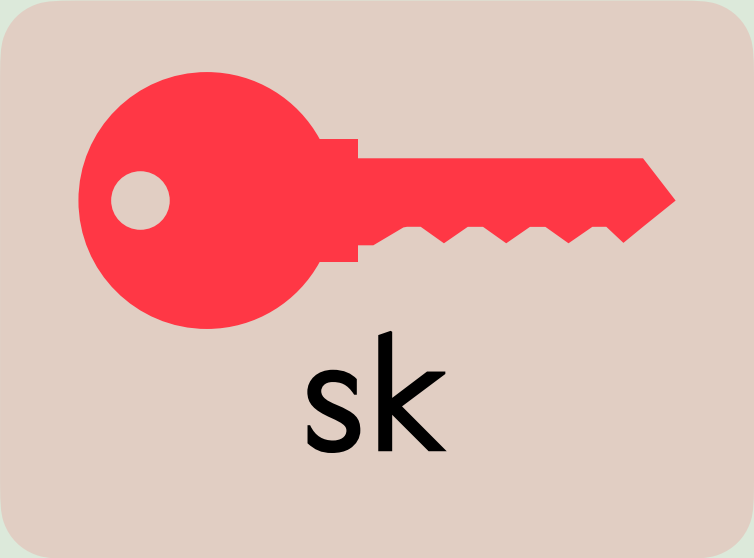
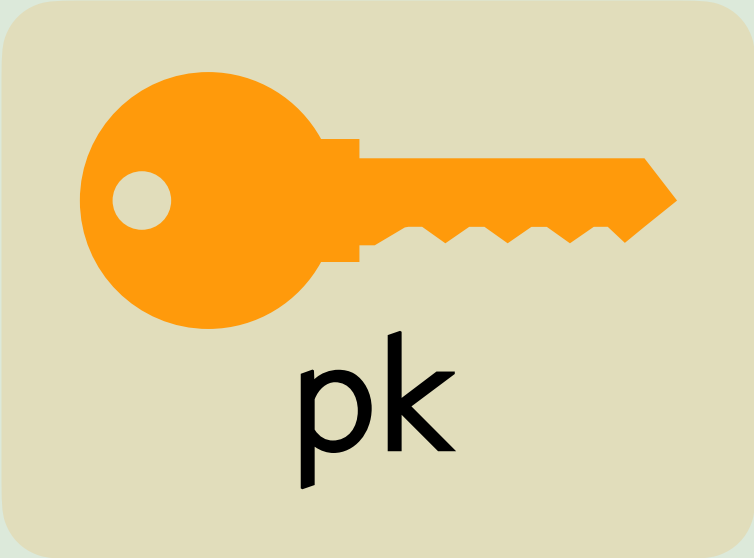
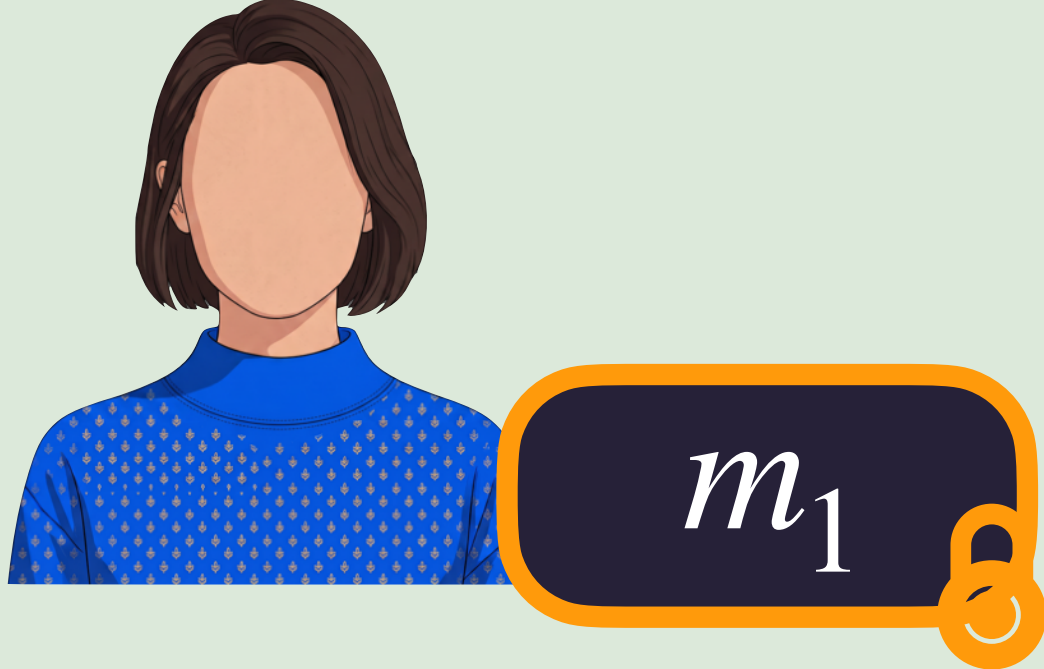
Index Dependence

ℓ : max batch size = 3



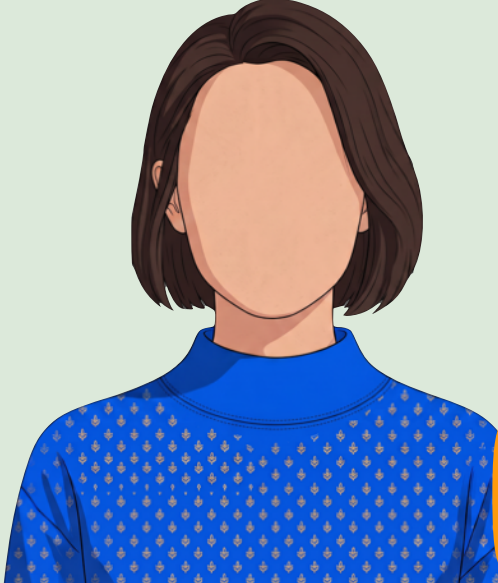
Index Dependence

ℓ : max batch size = 3



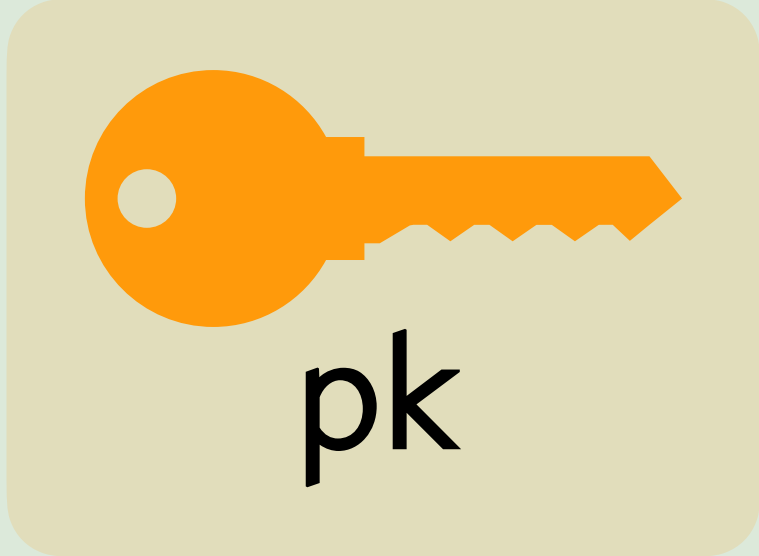
Index Dependence

ℓ : max batch size = 3

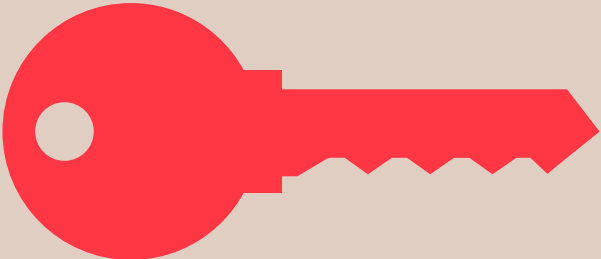


m_1

1







sk



m_2



m_3

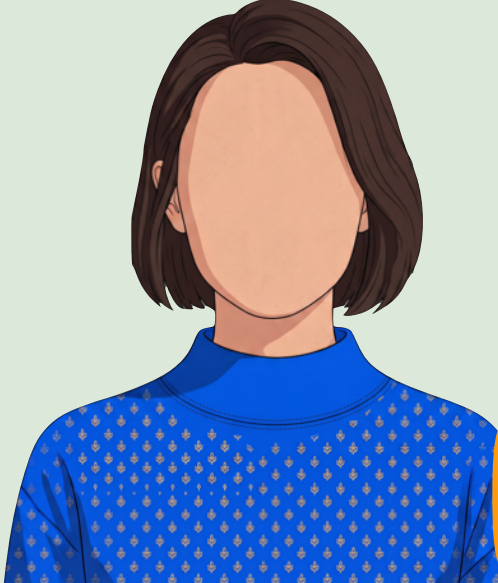
1:

2:

3:

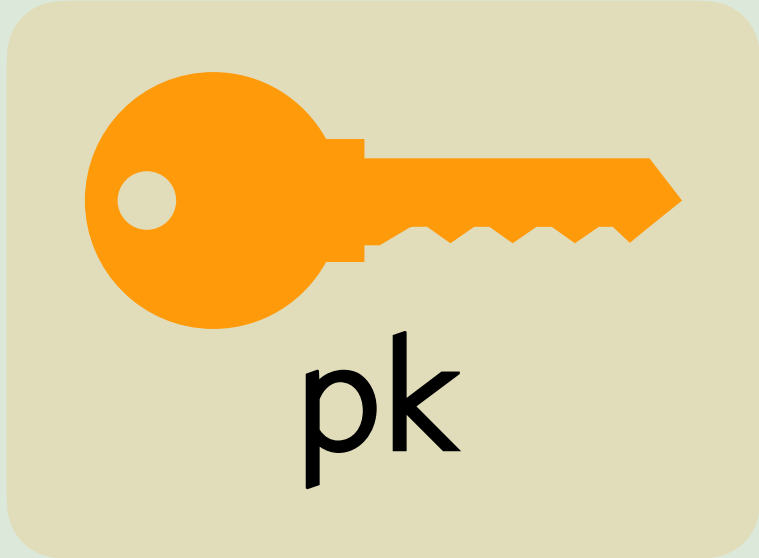
Index Dependence

ℓ : max batch size = 3



m_1

1





sk



m_2

2



m_3

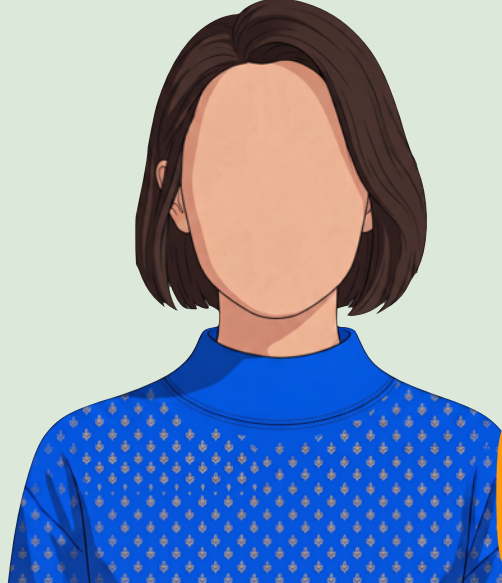
1:

2:

3:

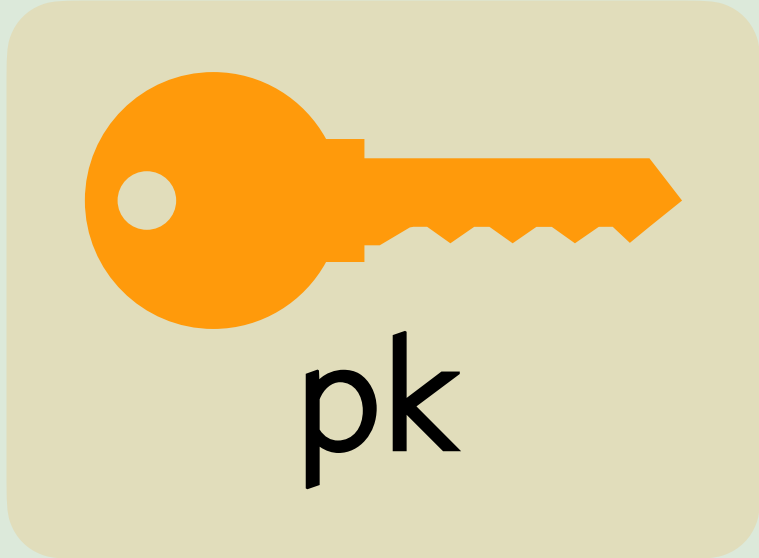
Index Dependence

ℓ : max batch size = 3



m_1

1





sk



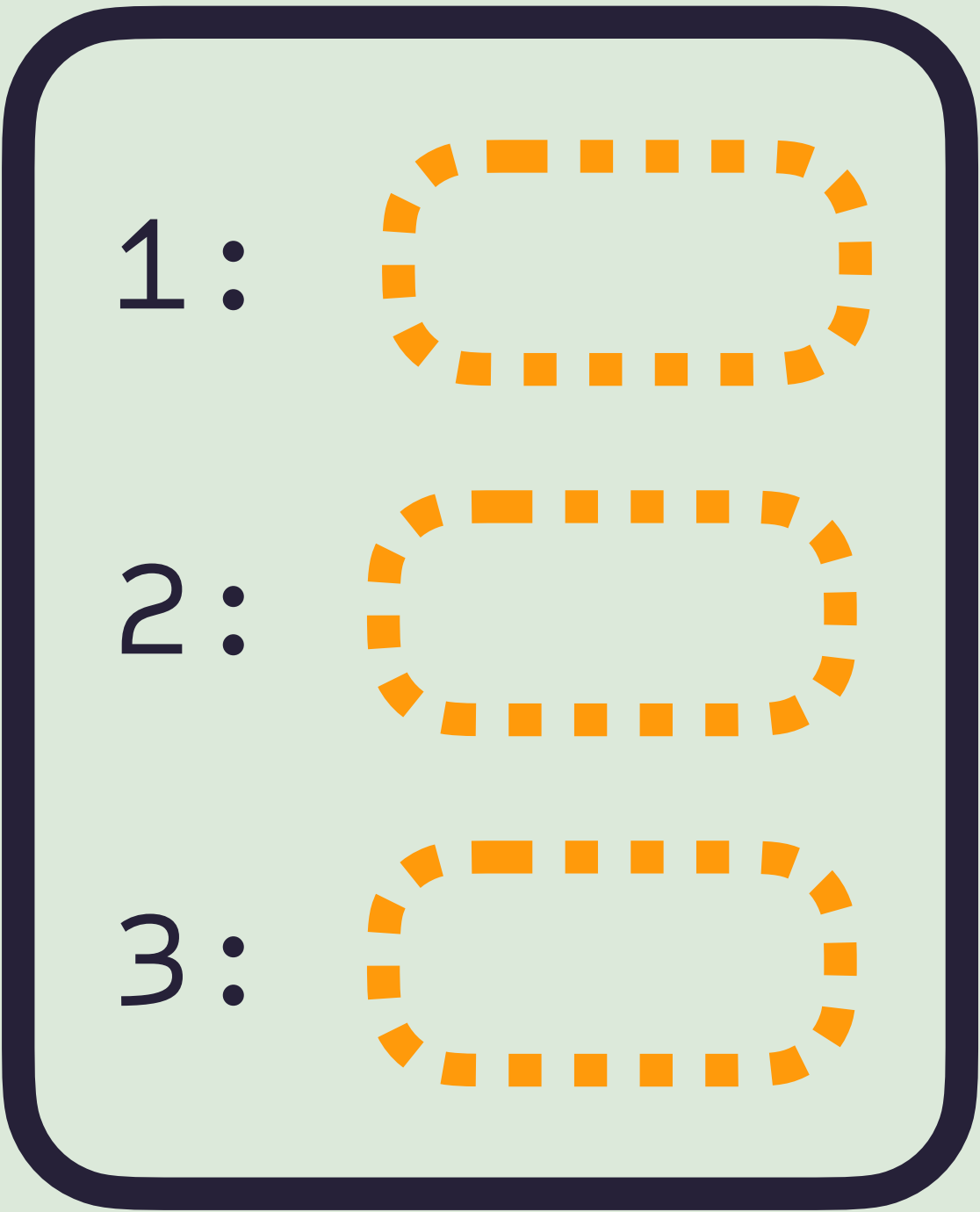
m_2

2



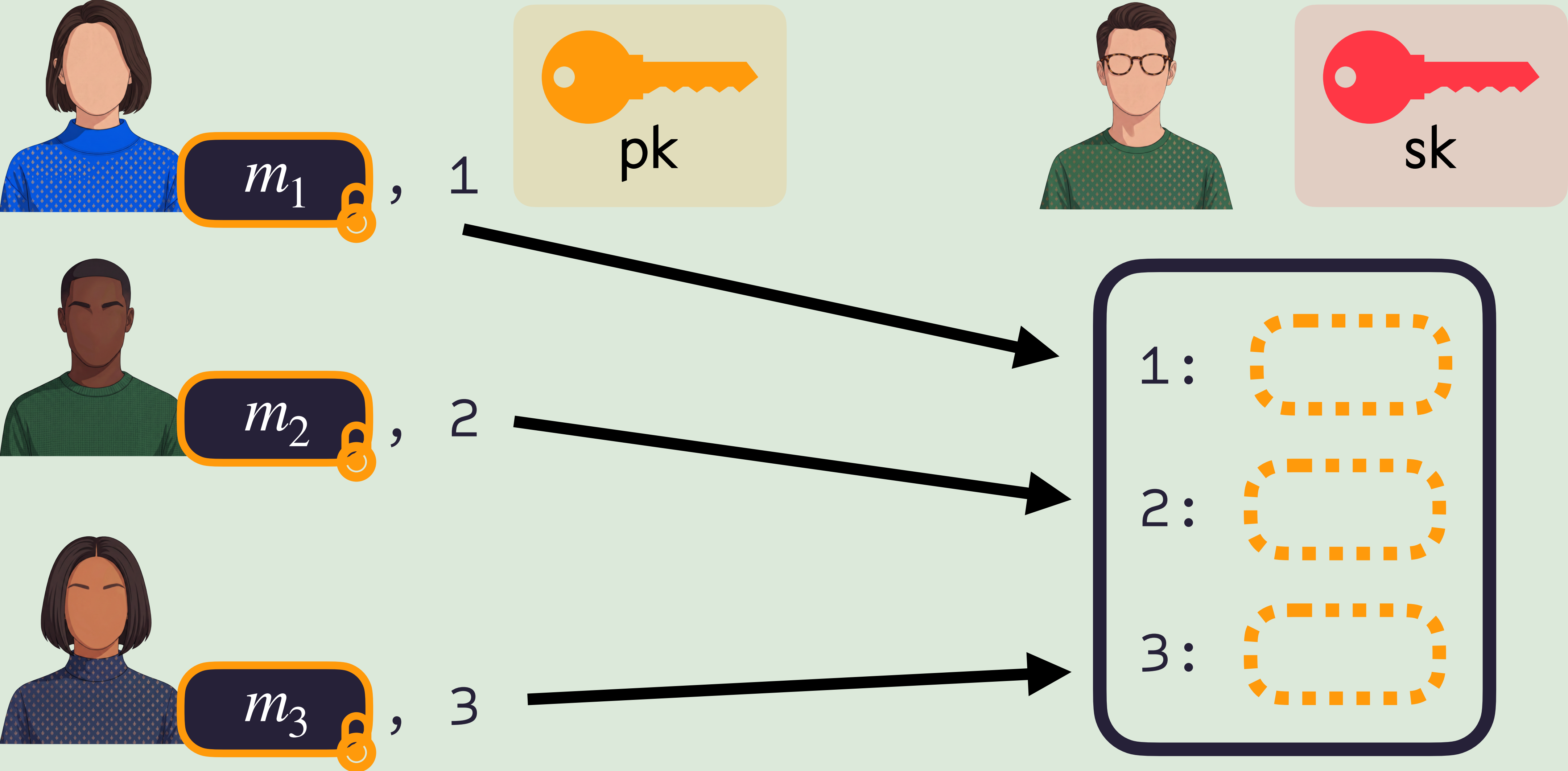
m_3

3



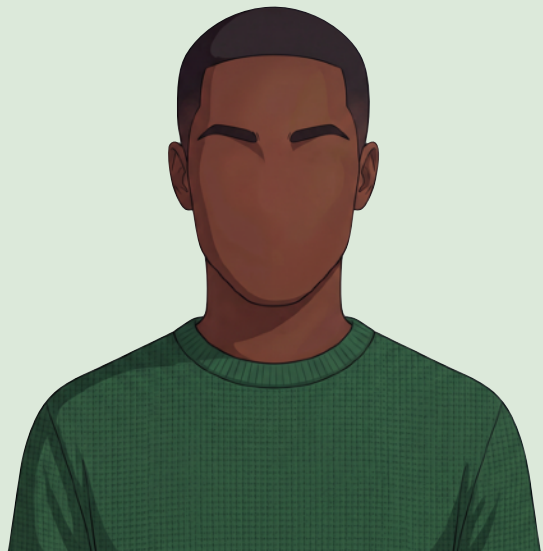
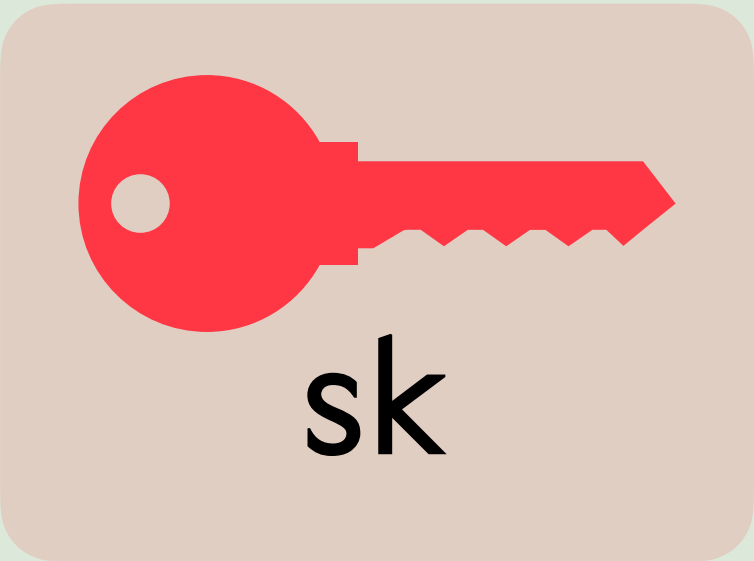
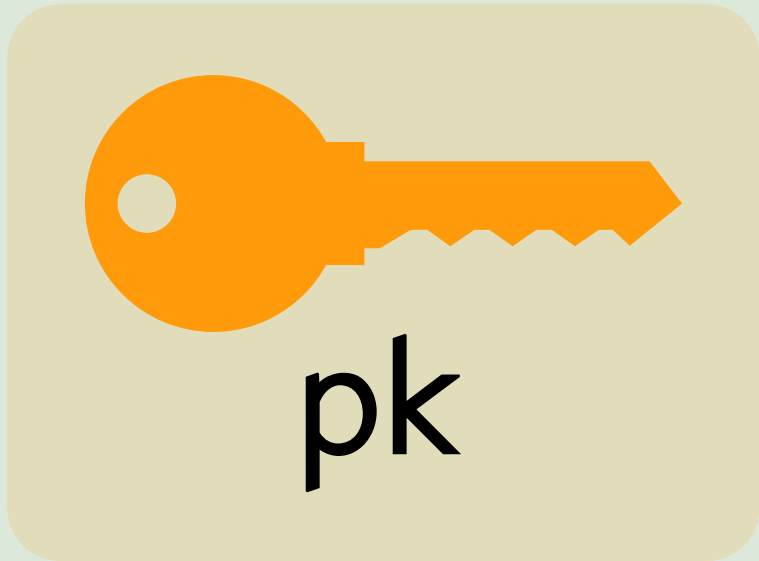
Index Dependence

ℓ : max batch size = 3



Index Dependence

ℓ : max batch size = 3



1:

m_1



2:

m_2



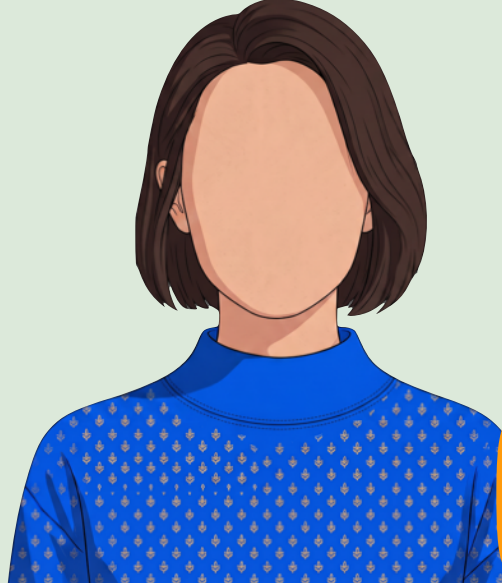
3:

m_3



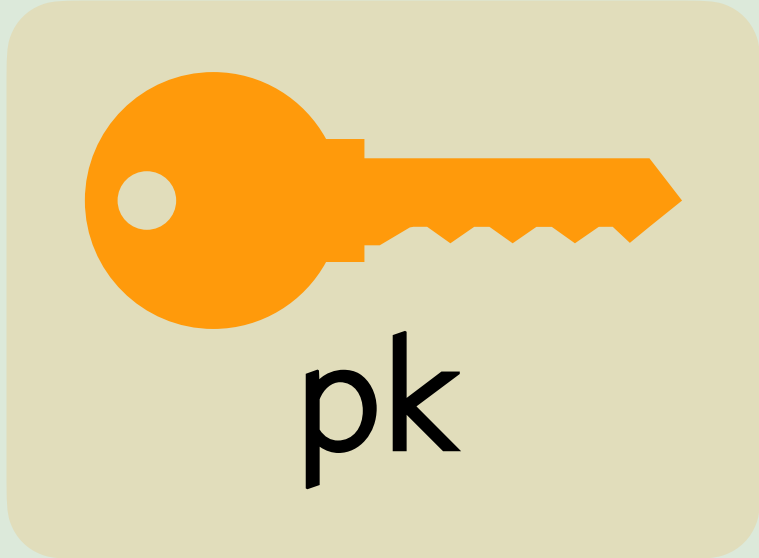
Index Dependence

ℓ : max batch size = 3



m_1

1





sk



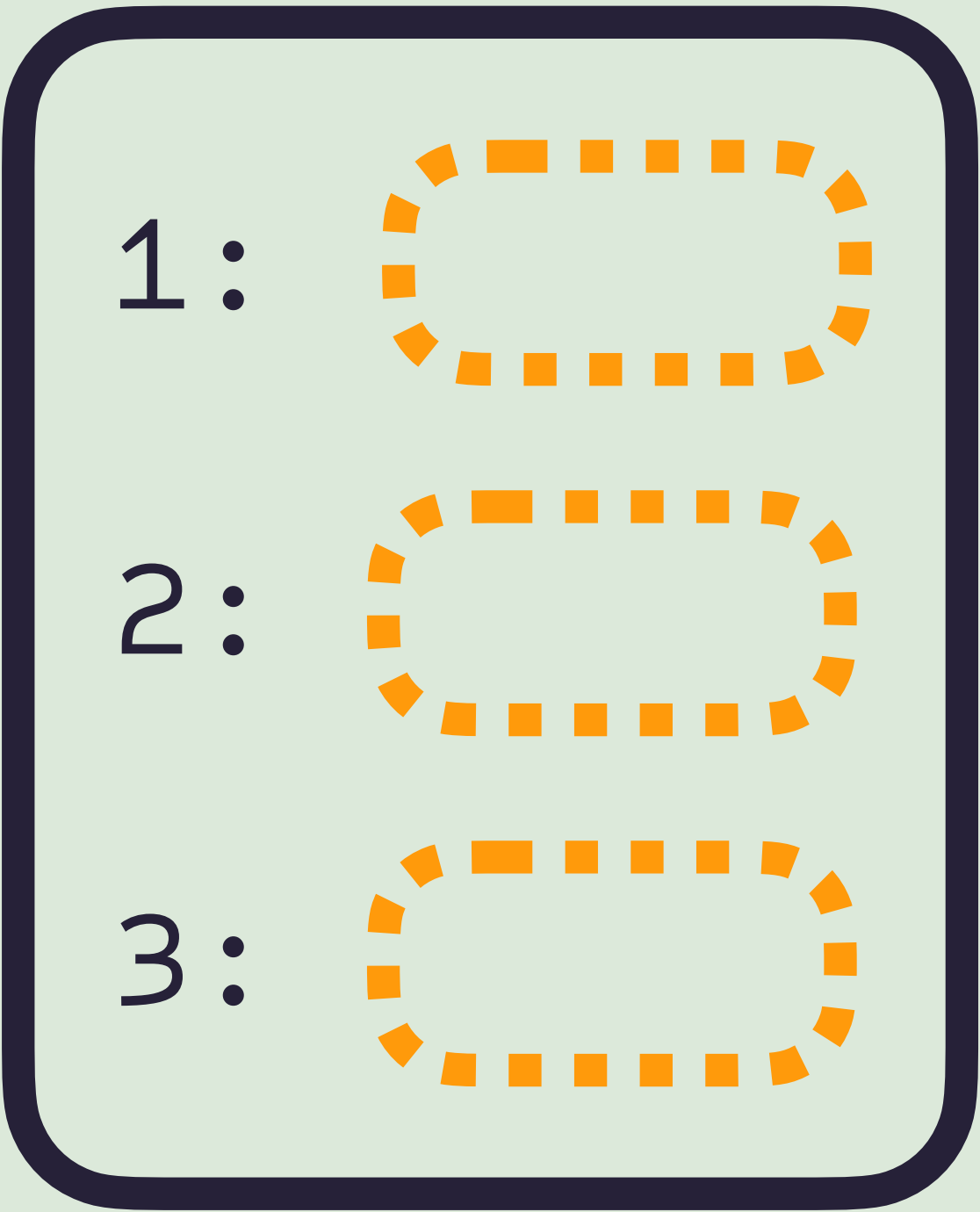
m_2

2



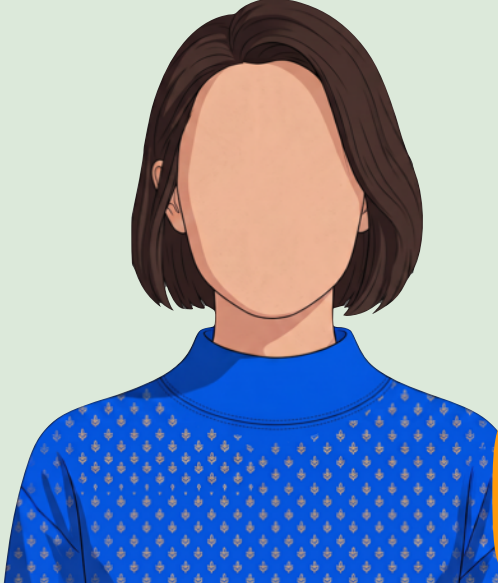
m_3

1



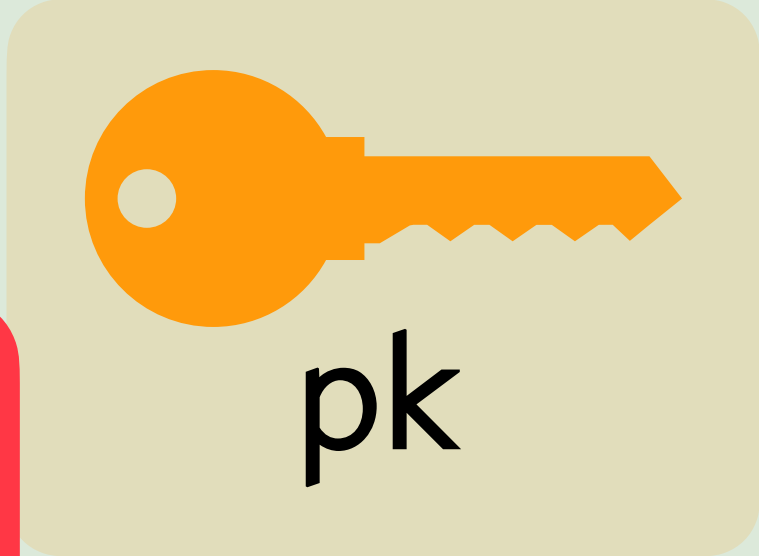
Index Dependence

ℓ : max batch size = 3



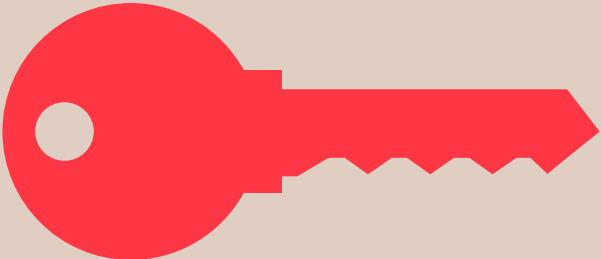
m_1

1



pk





sk



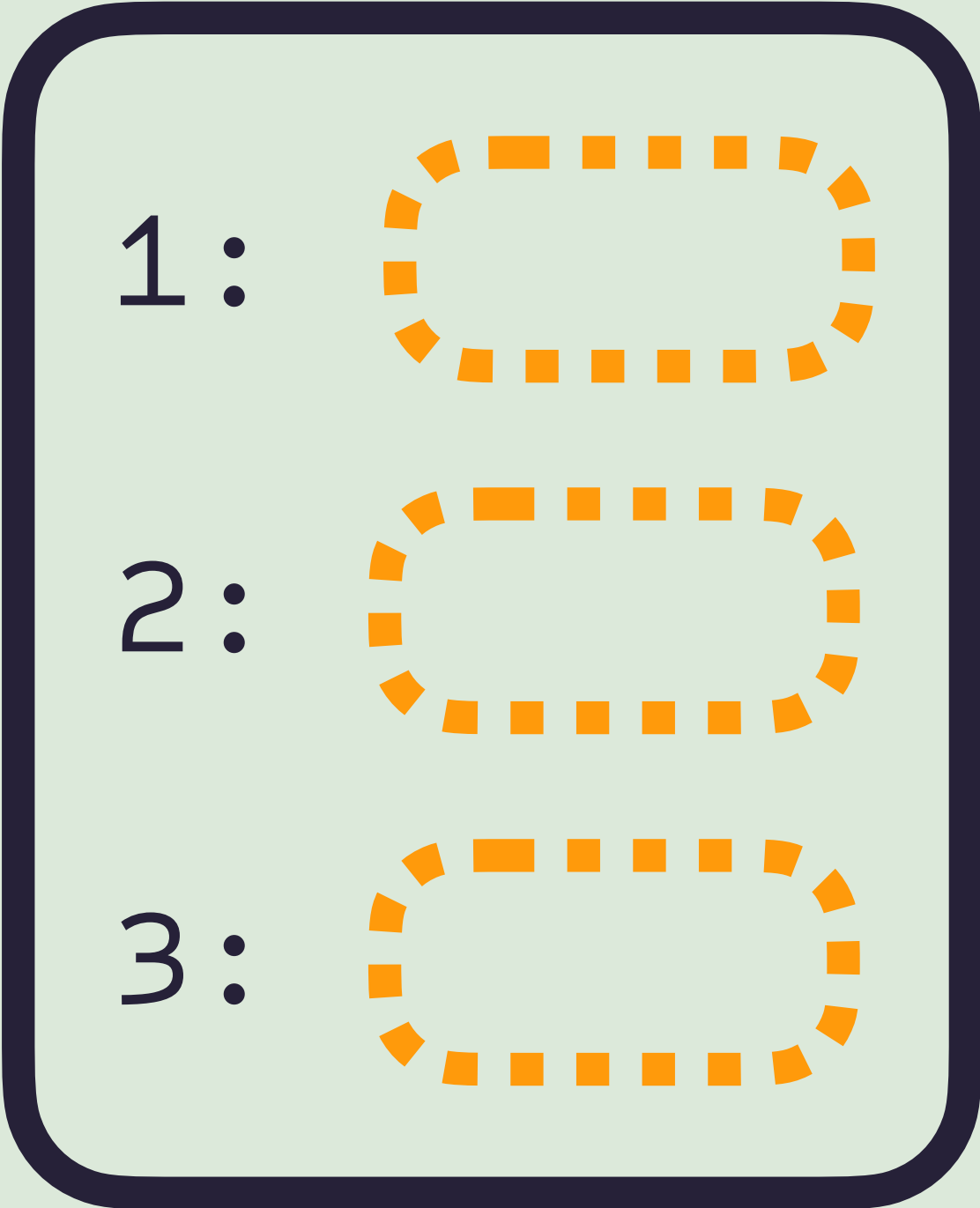
m_2

2



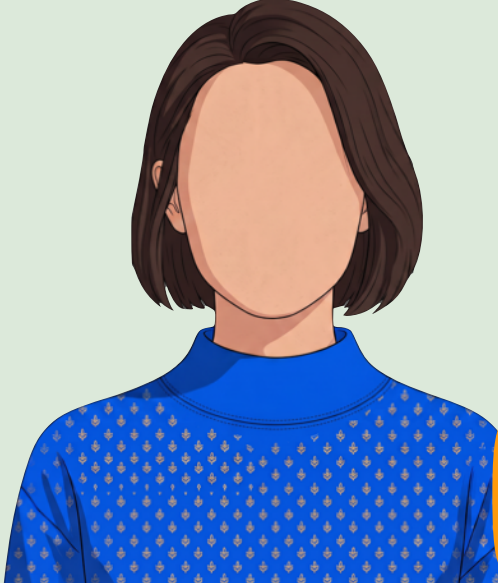
m_3

1



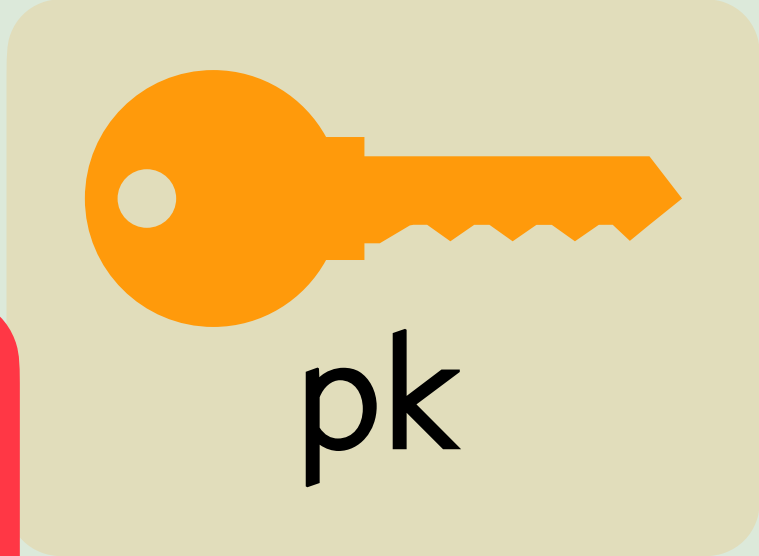
Index Dependence

ℓ : max batch size = 3



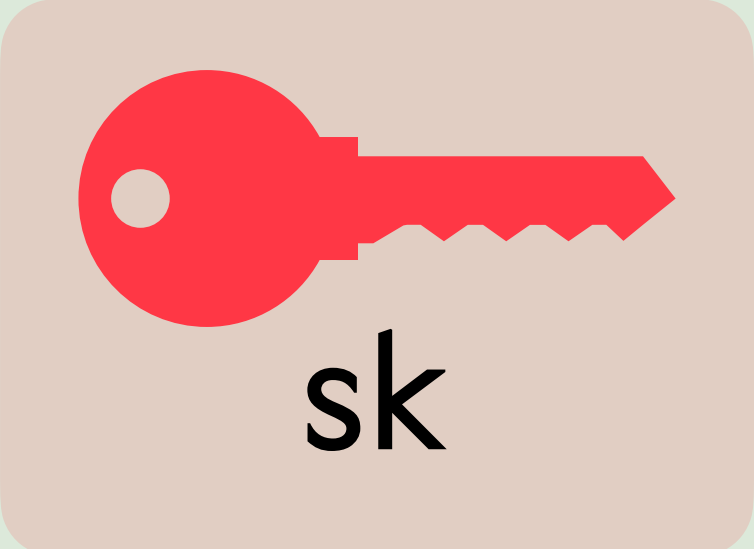
m_1

1



pk





sk



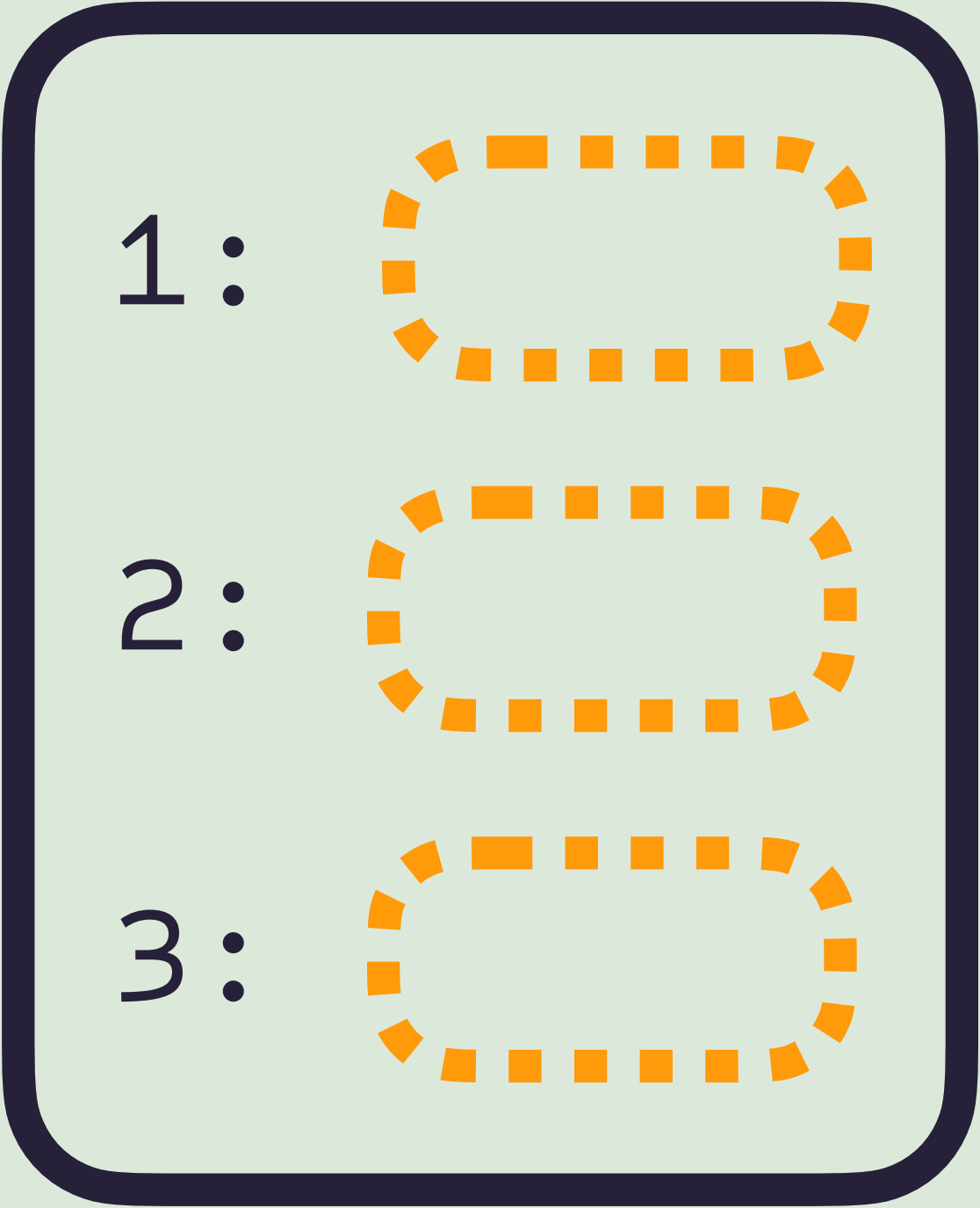
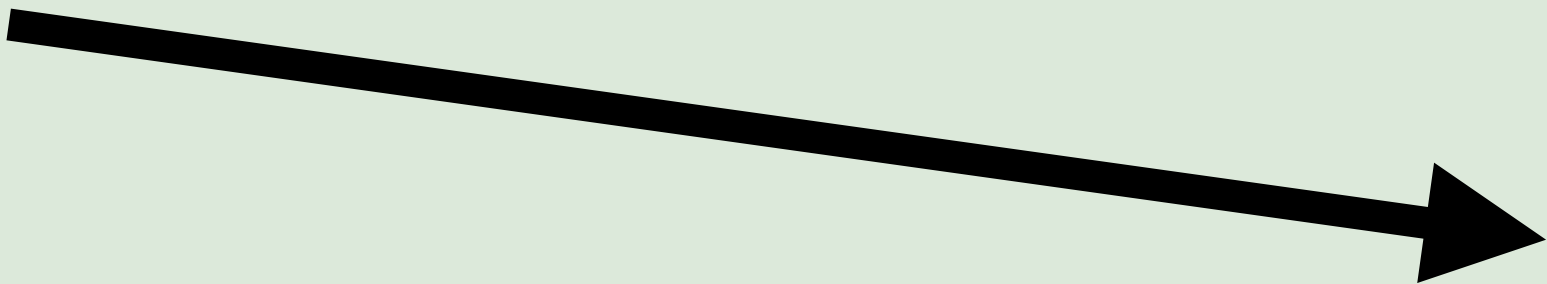
m_2

2



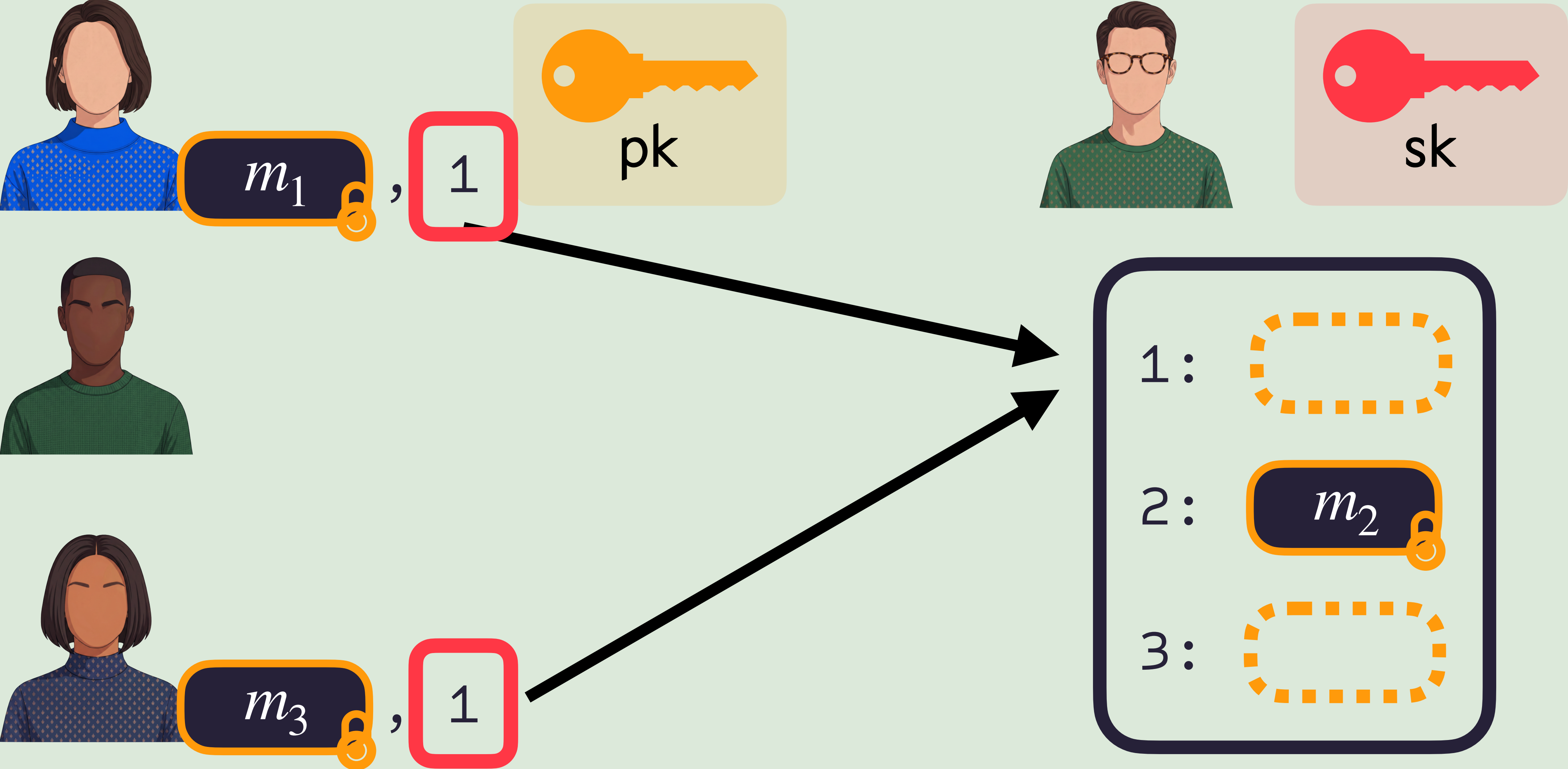
m_3

1



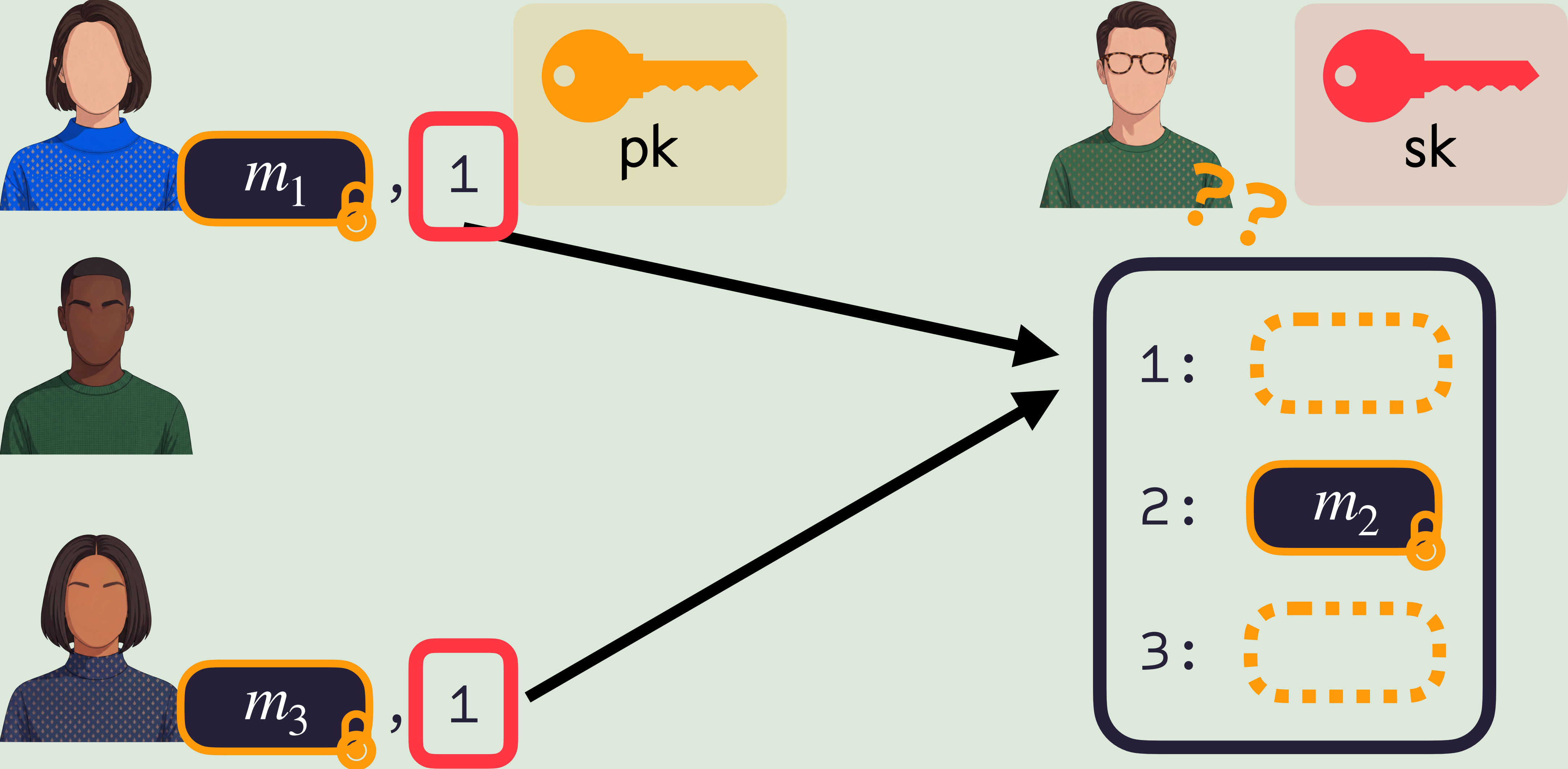
Index Dependence

ℓ : max batch size = 3



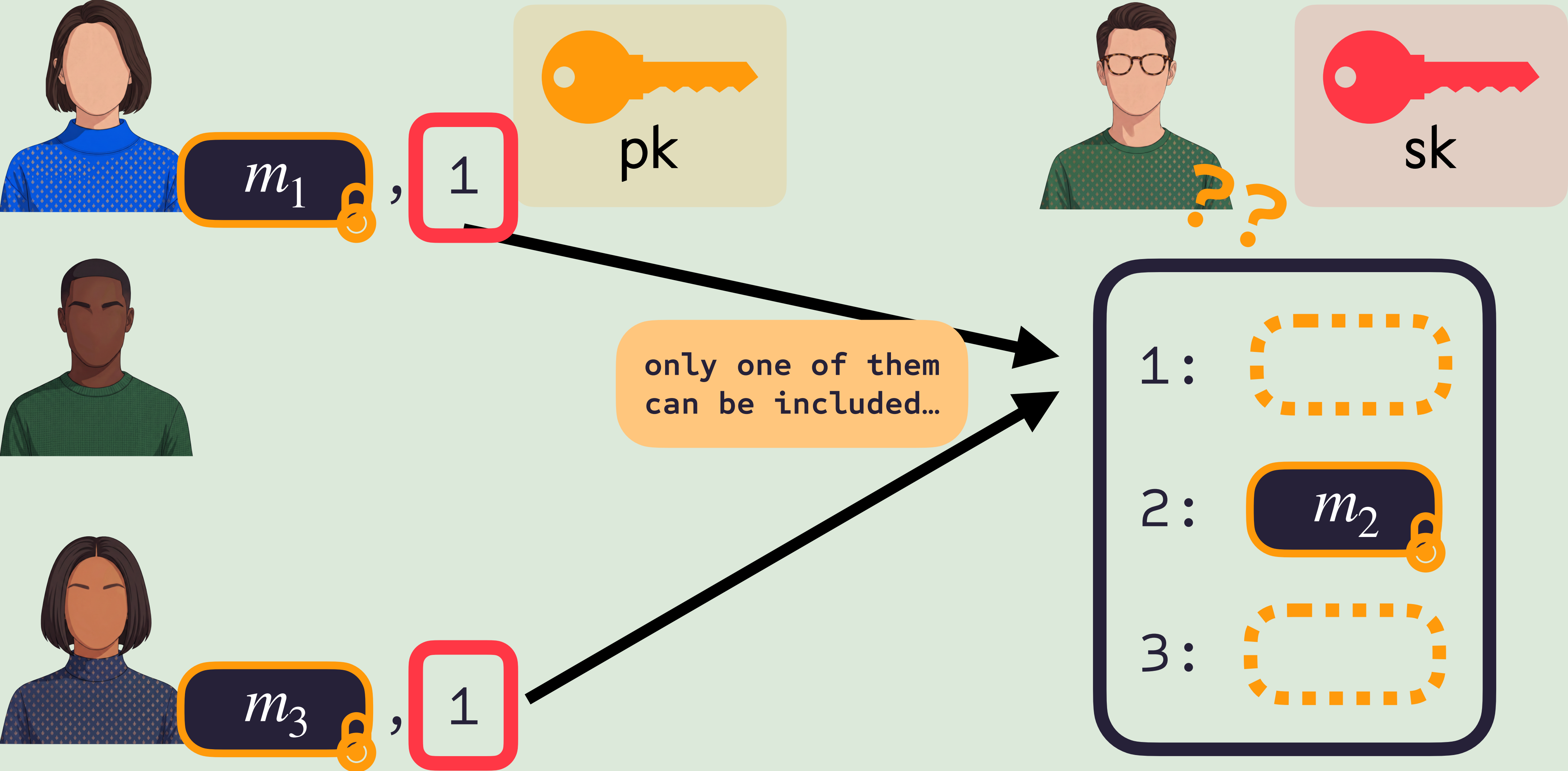
Index Dependence

ℓ : max batch size = 3



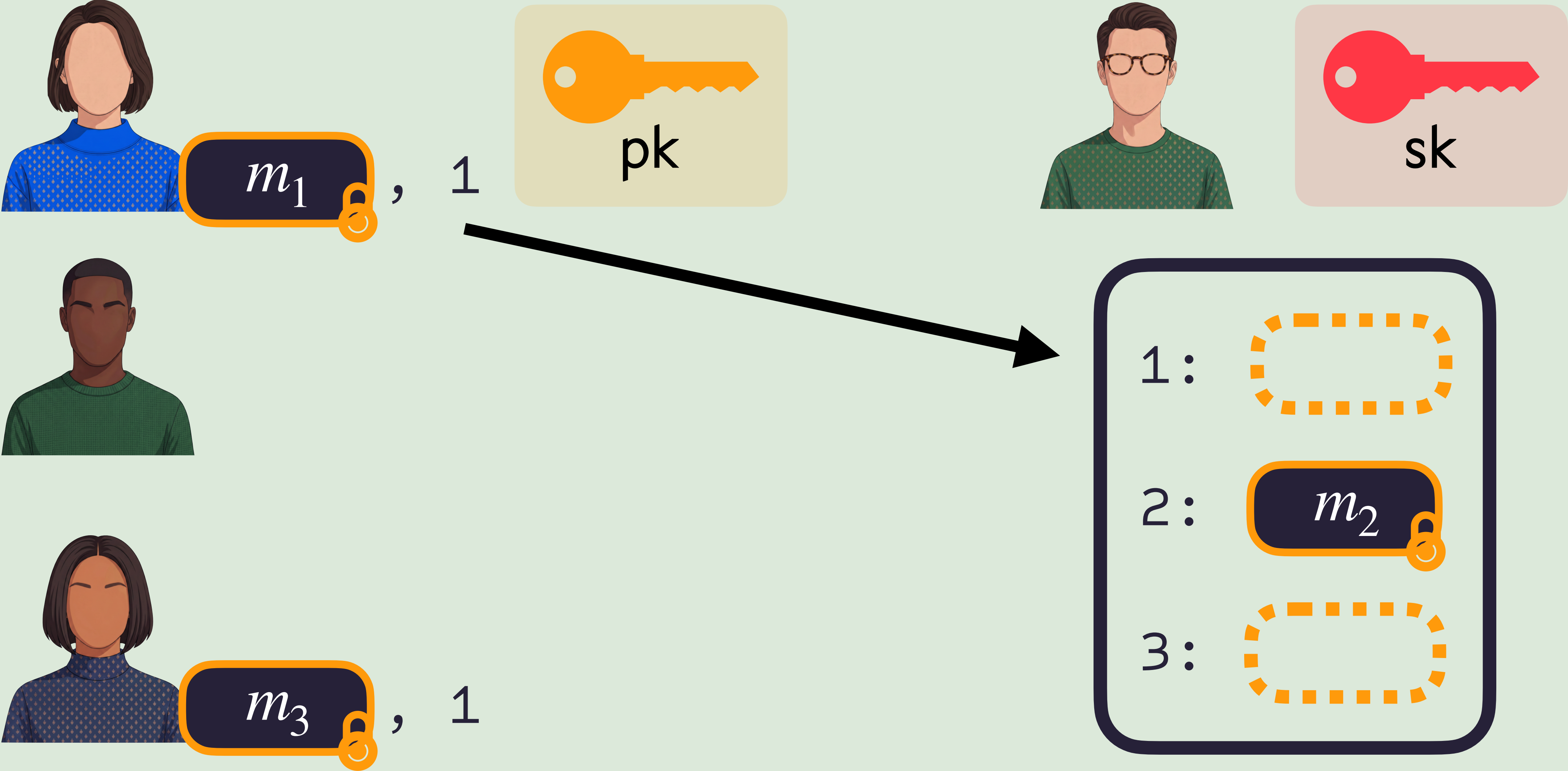
Index Dependence

ℓ : max batch size = 3



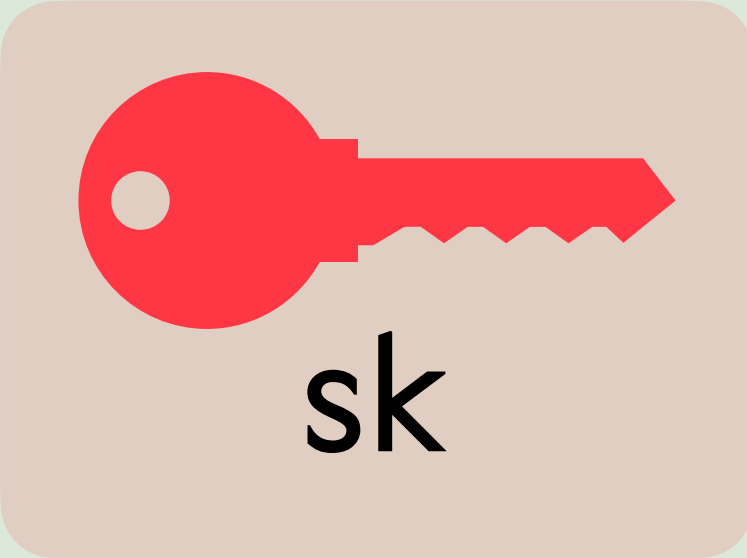
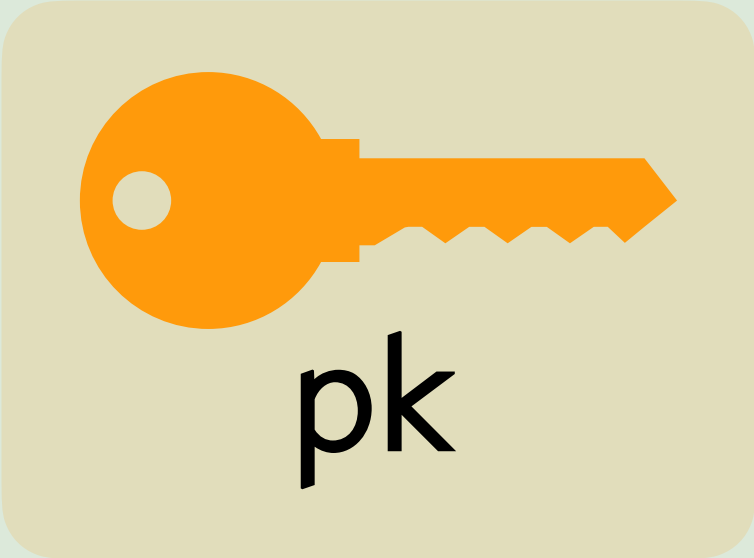
Index Dependence

ℓ : max batch size = 3



Index Dependence

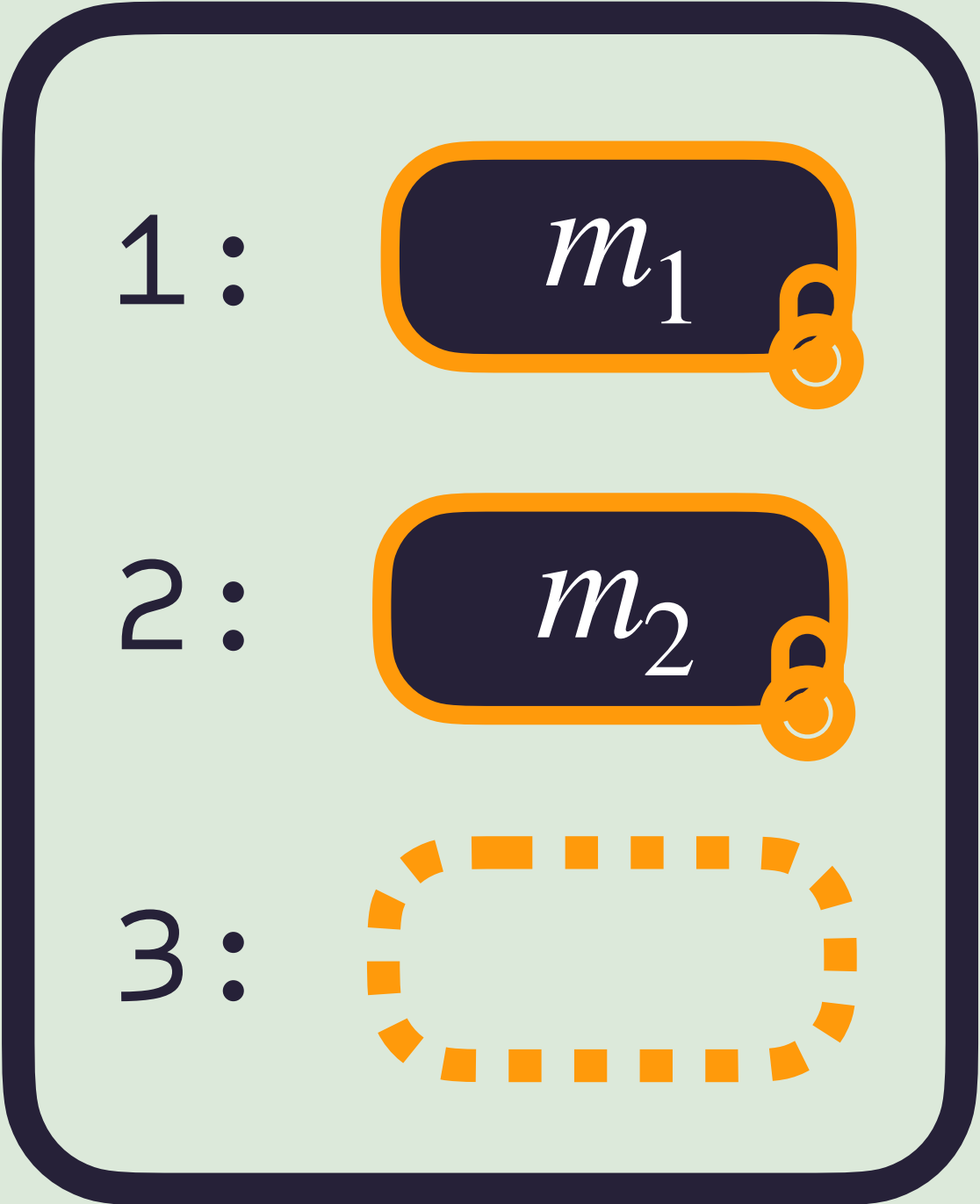
ℓ : max batch size = 3



sad.. there
was space for
me too!

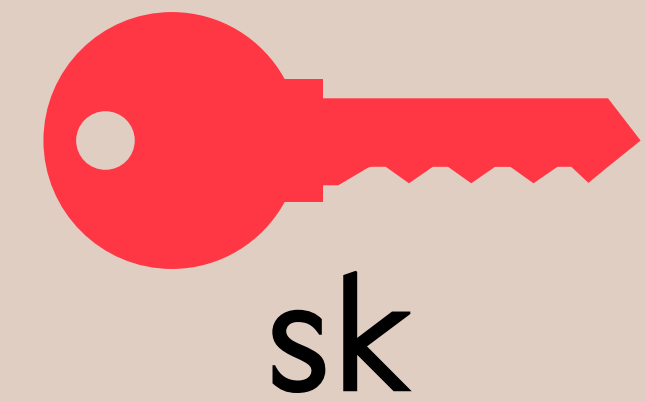
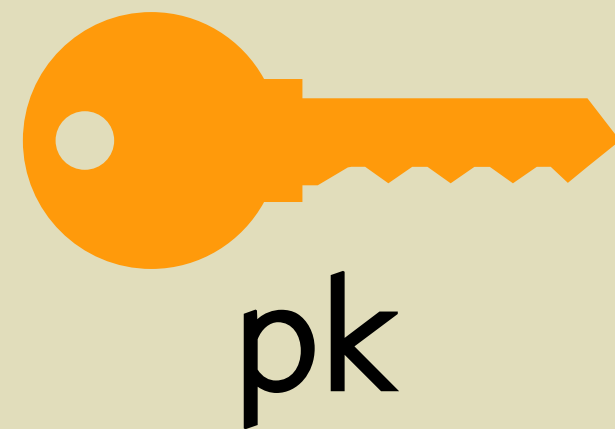


m_3 , 1



Index Dependence

ℓ : max batch size = 3



sad... there
was space for
me too!



m_3

, 1

1:

m_1

for the rest of this talk, we'll look
at this weaker notion.

See the paper on how we achieve batch
encryption without index dependence

Intuition

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{(X+j)^2}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{(X+k)^2}$

Intuition

All parties hold *linear* fractions in $\mathbb{G}_2$:

$$\frac{1}{X+j} \quad \forall j \in [\ell];$$

and *quadratic* fractions in $\mathbb{G}_T$:

$$\frac{1}{(X+j)^2} \quad \forall j \in [\ell].$$

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{(X+j)^2}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{(X+k)^2}$

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Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{(X+j)^2}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{(X+k)^2}$

Ciphertext for index $i \in [\ell]$

One-time pad message
using *quadratic fraction*
on the diagonal times a
random value:

$$m + \frac{r_i}{(X+i)^2}$$

Intuition

All parties hold *linear* fractions in $\mathbb{G}_2$:
 $\frac{1}{X+j} \ \forall j \in [\ell];$
and *quadratic* fractions in $\mathbb{G}_T$:
 $\frac{1}{(X+j)^2} \ \forall j \in [\ell].$

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{(X+j)^2}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{(X+k)^2}$

Ciphertext for index $i \in [\ell]$

One-time pad message
using *quadratic fraction*
on the diagonal times a
random value:

$$m + \frac{r_i}{(X+i)^2}$$

Pre-decrypting batch

The pre-decryptor
computes the pre-
decryption key:

$$\text{sbk} = \sum_{j \in [\ell]} \frac{r_j}{X+j}$$

Intuition

All parties hold *linear* fractions in $\mathbb{G}_2$:

$$\frac{1}{X+j} \quad \forall j \in [\ell];$$

and *quadratic* fractions in $\mathbb{G}_T$:

$$\frac{1}{(X+j)^2} \quad \forall j \in [\ell].$$

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{(X+j)^2}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{(X+k)^2}$

Ciphertext for index $i \in [\ell]$

One-time pad message using *quadratic fraction* on the diagonal times a random value:

$$m + \frac{r_i}{(X+i)^2}$$

Pre-decrypting batch

The pre-decry computes the decryption key

r_j is used to tie ciphertext to index j

$$\text{sbk} = \sum_{j \in [\ell]} \frac{r_j}{X+j}$$

Intuition

All parties hold *linear* fractions in $\mathbb{G}_2$:
 $\frac{1}{X+j} \forall j \in [\ell]$;
 and *quadratic* fractions in $\mathbb{G}_T$:
 $\frac{1}{(X+j)^2} \forall j \in [\ell]$.

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{X+i} \cdot \frac{1}{X+j}$	$\frac{1}{X+i} \cdot \frac{1}{X+k}$
$-j$	$\frac{1}{X+j} \cdot \frac{1}{X+i}$	$\frac{1}{(X+j)^2}$	$\frac{1}{X+j} \cdot \frac{1}{X+k}$
$-k$	$\frac{1}{X+k} \cdot \frac{1}{X+i}$	$\frac{1}{X+k} \cdot \frac{1}{X+j}$	$\frac{1}{(X+k)^2}$

Ciphertext for index $i \in [\ell]$

One-time pad message
 using *quadratic fraction*
 on the diagonal times a
 random value:

$$m + \frac{r_i}{(X+i)^2}$$

Pre-decrypting batch

The pre-decry
 computes the
 decryption key

$$\text{sbk} = \sum_{j \in [\ell]} \frac{r_j}{X+j}$$

r_j is used to tie
 ciphertext to index j

Decryption for index $i \in [\ell]$

Compute $\text{sbk} \cdot \left(\frac{1}{X+i} \right)$

$$= \frac{r_i}{(X+i)^2} + \sum_{j \neq i} \frac{r_i}{(X+j)} \cdot \frac{1}{(X+i)}$$

Intuition

All parties hold *linear* fractions in $\mathbb{G}_2$:

$$\frac{1}{X+j} \quad \forall j \in [\ell];$$

and *quadratic* fractions in $\mathbb{G}_T$:

$$\frac{1}{(X+j)^2} \quad \forall j \in [\ell].$$

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$
$-j$	$\frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$	$\frac{1}{(X+j)^2}$	$\frac{1}{k-j} \left(\frac{1}{X+k} - \frac{1}{X+j} \right)$
$-k$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$	$\frac{1}{k-j} \left(\frac{1}{X+k} - \frac{1}{X+j} \right)$	$\frac{1}{(X+k)^2}$

Ciphertext for index $i \in [\ell]$

One-time pad message using **quadratic fraction** on the diagonal multiplied by a random value:

$$m + \frac{r_i}{(X+i)^2}$$

Pre-decrypting batch

The pre-decryptor computes the pre-decryption key:

$$\text{sbk} = \sum_{j \in [\ell]} \frac{r_j}{X+j}$$

Decryption for index $i \in [\ell]$

$$\begin{aligned} &\text{Compute } \text{sbk} \cdot \left(\frac{1}{X+i} \right) \\ &= \frac{r_i}{(X+i)^2} + \sum_{j \neq i} \frac{r_j}{(X+j)} \cdot \frac{1}{(X+i)} \end{aligned}$$

Intuition

All parties hold *linear* fractions in $\mathbb{G}_2$:
 $\frac{1}{X+j} \forall j \in [\ell]$;
 and *quadratic* fractions in $\mathbb{G}_T$:
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Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$
$-j$	$\frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$	$\frac{1}{(X+j)^2}$	$\frac{1}{k-j} \left(\frac{1}{X+k} - \frac{1}{X+j} \right)$
$-k$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$	$\frac{1}{k-j} \left(\frac{1}{X+k} - \frac{1}{X+j} \right)$	$\frac{1}{(X+k)^2}$

Ciphertext for index $i \in [\ell]$

One-time pad message using **quadratic fraction** on the diagonal multiplied by a random value:

$$m + \frac{r_i}{(X+i)^2}$$

Pre-decrypting batch

The pre-decryptor computes the pre-decryption key:

$$\text{sbk} = \sum_{j \in [\ell]} \frac{r_j}{X+j}$$

Decryption for index $i \in [\ell]$

Compute $\text{sbk} \cdot \left(\frac{1}{X+i} \right)$

$$= \frac{r_i}{(X+i)^2} + \sum_{j \neq i} \frac{r_j}{(X+j)} \cdot \frac{1}{(X+i)}$$

off-diagonal products linearize and can be subtracted out

Intuition

All parties hold *linear* fractions in $\mathbb{G}_2$:

$$\frac{1}{X+j} \quad \forall j \in [\ell];$$

and *quadratic* fractions in $\mathbb{G}_T$:

$$\frac{1}{(X+j)^2} \quad \forall j \in [\ell].$$

Poles	$-i$	$-j$	$-k$
$-i$	$\frac{1}{(X+i)^2}$	$\frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$
$-j$	$\frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right)$	$\frac{1}{(X+j)^2}$	$\frac{1}{k-j} \left(\frac{1}{X+k} - \frac{1}{X+j} \right)$
$-k$	$\frac{1}{k-i} \left(\frac{1}{X+i} - \frac{1}{X+k} \right)$	$\frac{1}{k-j} \left(\frac{1}{X+k} - \frac{1}{X+j} \right)$	$\frac{1}{(X+k)^2}$

Ciphertext for index $i \in [\ell]$

One-time pad message using **quadratic fraction** on the diagonal multiplied by a random value:

$$m + \frac{r_i}{(X+i)^2}$$

Pre-decrypting batch

The pre-decryptor computes the pre-decryption key:

$$\text{sbk} = \sum_{j \in [\ell]} \frac{r_j}{X+j}$$

Decryption for index $i \in [\ell]$

Compute $\text{sbk} \cdot \left(\frac{1}{X+i} \right)$

$$= \frac{r_i}{(X+i)^2} - \sum_{j \neq i} \frac{r_j}{(X+j)} \cdot \frac{1}{(X+i)}$$

and then you're just left with the one-time pad!

The Index-Dependent Construction

ℓ : max block size

Recall (asymmetric) bilinear group: $\mathcal{G} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$;

additive notation: $[x]_1 = x \cdot g_1$, $[y]_2 = y \cdot g_2$, and $[z]_T = z \cdot e(g_1, g_2)$

The Index-Dependent Construction

ℓ : max block size

Recall (asymmetric) bilinear group: $\mathcal{G} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$;

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Setup(1^λ , batch size 1^ℓ)

Sample $x \xleftarrow{\$} \mathbb{Z}_p$, and output

$\text{sk} := x$, and $\text{pk} :=$

(1) linear fractions: $\left[\frac{1}{x+1} \right]_2, \dots, \left[\frac{1}{x+\ell} \right]_2$, and

(2) quadratic fractions: $\left[\frac{1}{(x+1)^2} \right]_T, \dots, \left[\frac{1}{(x+\ell)^2} \right]_T$

The Index-Dependent Construction

ℓ : max block size

Recall (asymmetric) bilinear group: $\mathcal{G} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$;

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Setup(1^λ , batch size 1^ℓ)

Sample $x \xleftarrow{\$} \mathbb{Z}_p$, and output

$\text{sk} := x$, and $\text{pk} :=$

(1) linear fractions: $\left[\frac{1}{x+1}\right]_2, \dots, \left[\frac{1}{x+\ell}\right]_2$, and

(2) quadratic fractions: $\left[\frac{1}{(x+1)^2}\right]_T, \dots, \left[\frac{1}{(x+\ell)^2}\right]_T$

Enc(pk, index $i \in [\ell]$, msg m)

Sample $r \xleftarrow{\$} \mathbb{Z}_p$, and output
ciphertext with two components

(1) $[r]_1$ (2) $r \left[\frac{1}{(x+i)^2}\right]_T + m$

The Index-Dependent Construction

ℓ : max block size

Recall (asymmetric) bilinear group: $\mathcal{G} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$;

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Setup(1^λ , batch size 1^ℓ)

Sample $x \xleftarrow{\$} \mathbb{Z}_p$, and output

$\text{sk} := x$, and $\text{pk} :=$

(1) linear fractions: $\left[\frac{1}{x+1} \right]_2, \dots, \left[\frac{1}{x+\ell} \right]_2$, and

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needed for
CCA-2 security

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PreDec(sk, $\{ct_i\}_i$)

Check NIZK proofs, and if valid, **output pre-decryption key as sum of partial fractions:**

$$\text{sbk} := \sum_{i \in [\ell]} \frac{1}{x + i} \cdot ct_i[1] = \sum_{i \in [\ell]} \left[\frac{r_i}{x + i} \right]_1$$

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subtract out linear terms

Prior Work Limitations

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Ciphertext is tied to an index

Users must **encrypt to an index/position** in the block/batch. Thus, they **need to coordinate** and know their **position ahead of time**.

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“Index-dependent” Batch Encryption

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“Epoch-based”

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Large CRS

Index-independent and *epoch-less* schemes suffer from a large exponential-size CRS

Our Work

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Best of all worlds!

We achieve batch encryption that is

1. **Independent of index:** no coordination required
2. **Epochless:** pending messages don't need to be re-encrypted
3. **Much shorter practical CRS:** linear in batch size
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linear in the maximum size of the batch.

Shortly after our eprint was out, two subsequent works
[Pol26, ADG+26]
achieved the same using a different technique

Implementation

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Prototypical implementation (not very optimized) using arkworks:
BLS12-381, single core Apple M4 Pro, 24 GB memory

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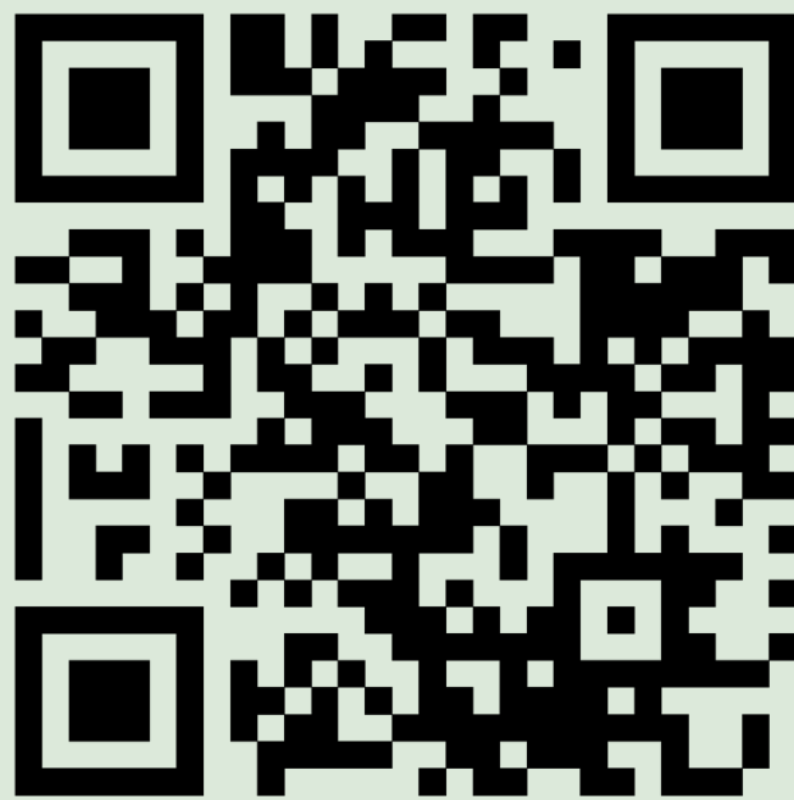
Batch size ℓ	Enc		PreDec		Dec	
	BE	BE _{short}	BE	BE _{short}	BE	BE _{short}
4	599 μ s	437 μ s	1.53 ms	0.82 ms	19.2 ms	15.3 ms
16	599 μ s	437 μ s	2.83 ms	1.93 ms	101 ms	76.3 ms
64	599 μ s	437 μ s	8.12 ms	6.93 ms	501 ms	364 ms
128	599 μ s	437 μ s	14.5 ms	13.2 ms	1.09 s	787 ms
256	599 μ s	437 μ s	27.9 ms	25.6 ms	2.38 s	1.69 s
512	599 μ s	437 μ s	54.3 ms	50.1 ms	5.12 s	3.61 s

Takeaways

Partial Fraction Techniques

Allows you to **linearize cross-terms**: $\frac{1}{X+i} \cdot \frac{1}{X+j} = \frac{1}{j-i} \left(\frac{1}{X+i} - \frac{1}{X+j} \right),$

and **mask values with higher-order poles**: $\frac{1}{(X+k)^2}$



introduces another interesting property using Cauchy matrices

Part 1
Partial Fraction Techniques
for Cryptography (2025/2081)



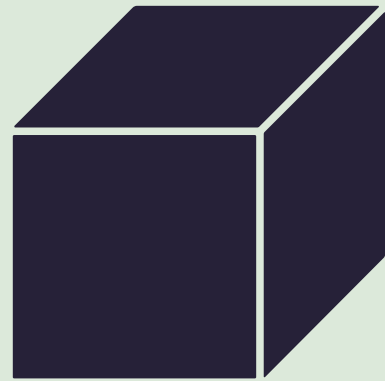
this paper

Part 2
Efficient Batch Threshold Encryption
Using Partial Fraction Techniques (2026/674)

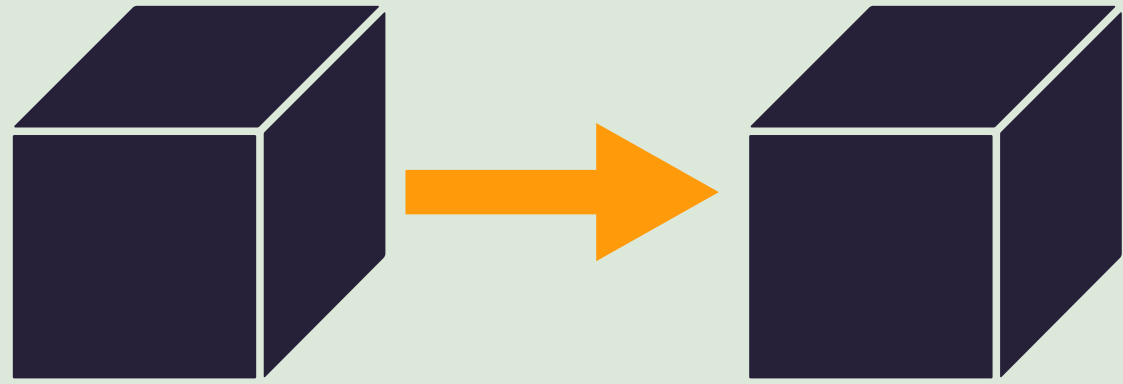
Appendix

Application: *Encrypted Mempools* for Blockchain

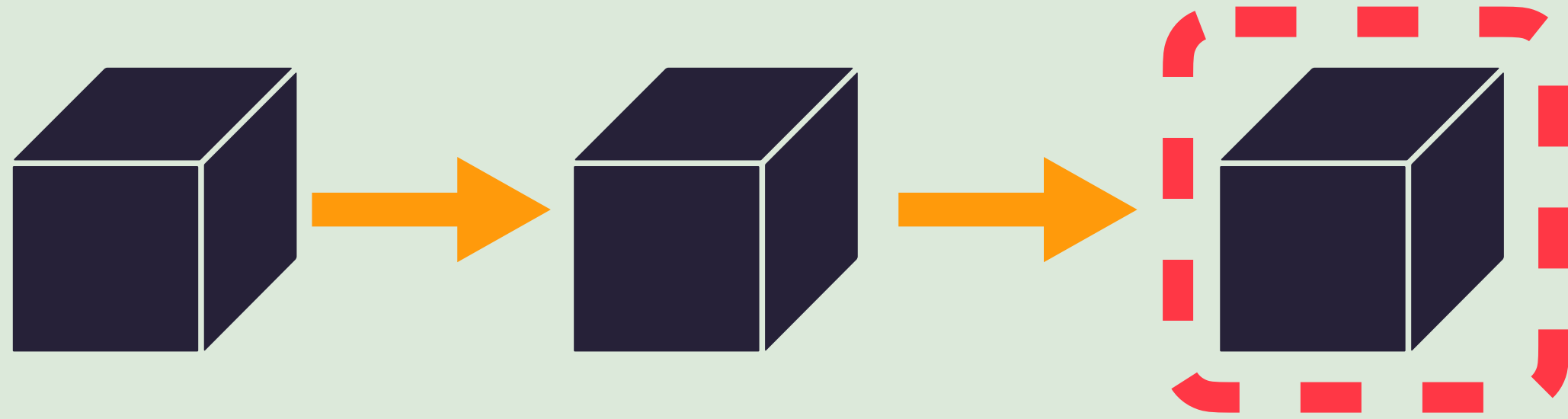
Application: *Encrypted Mempools* for Blockchain



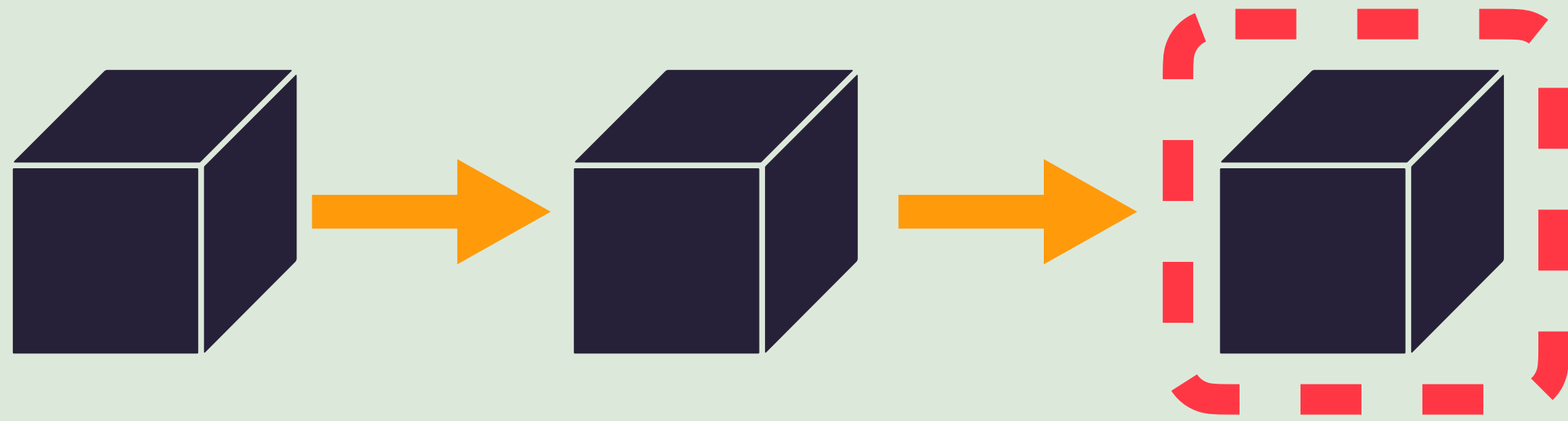
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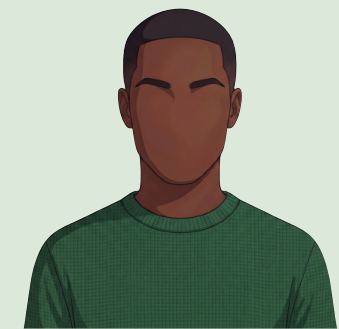
Application: *Encrypted Mempools* for Blockchain



users with transactions



tx₁



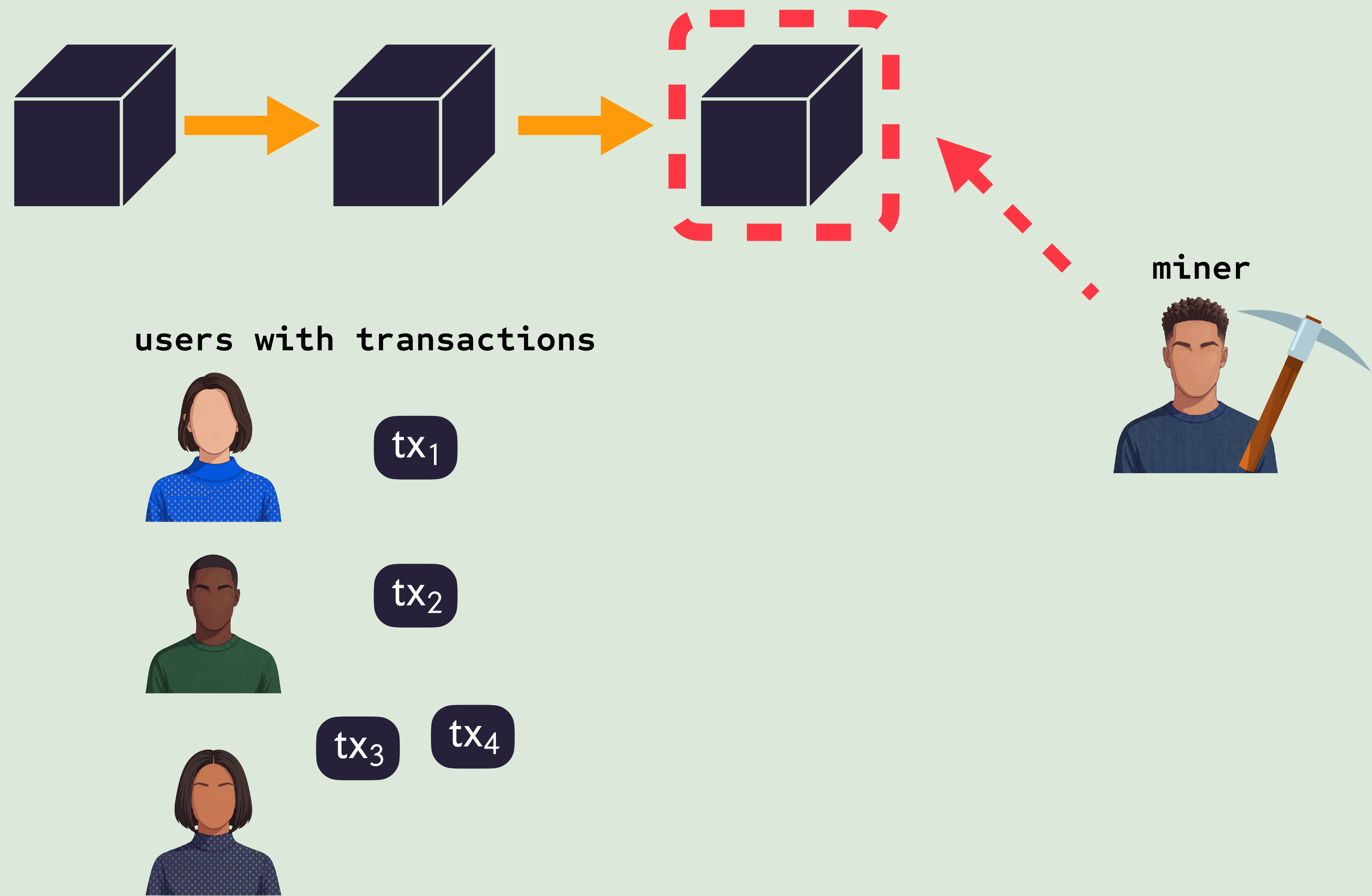
tx₂



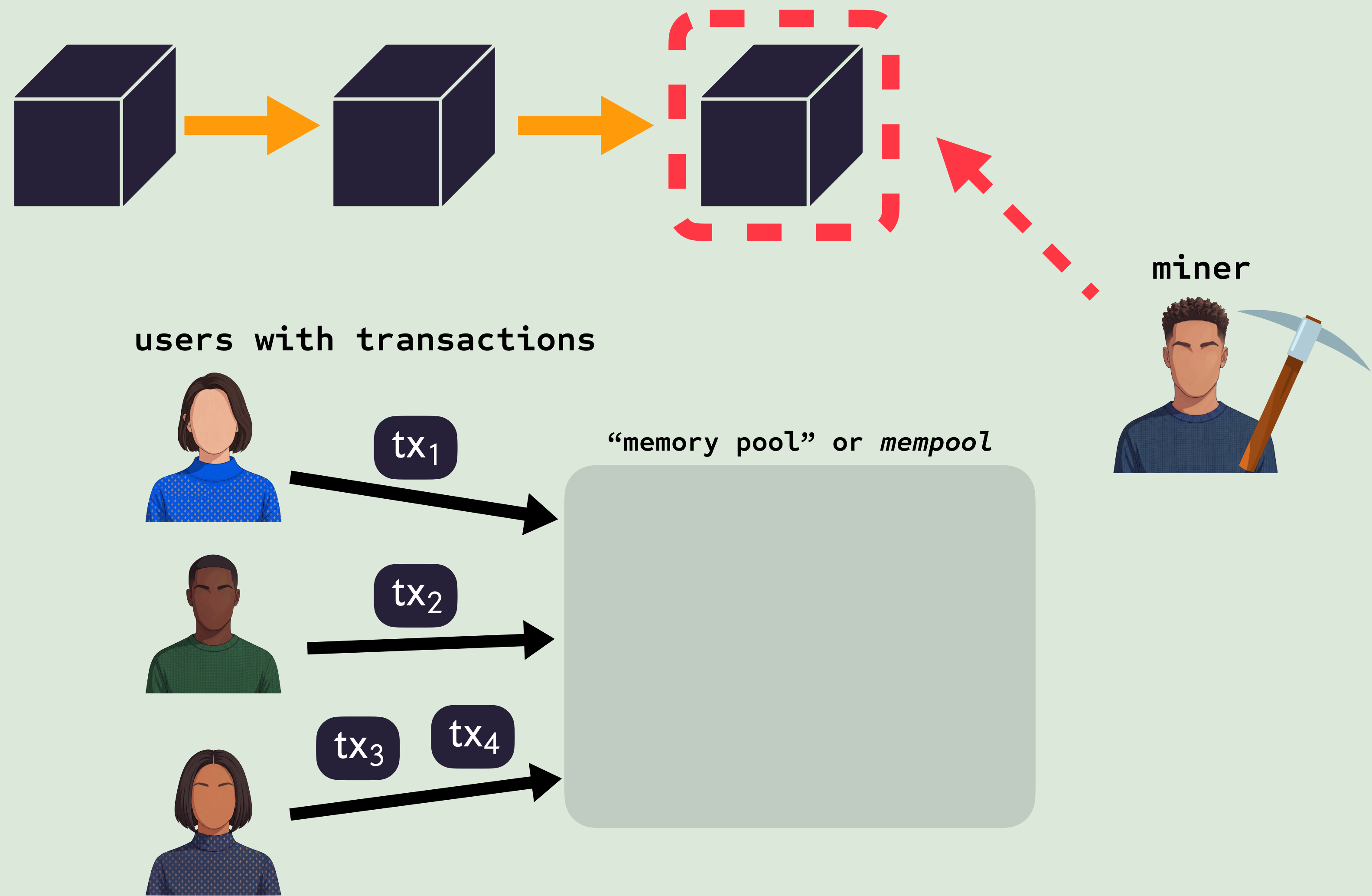
tx₃

tx₄

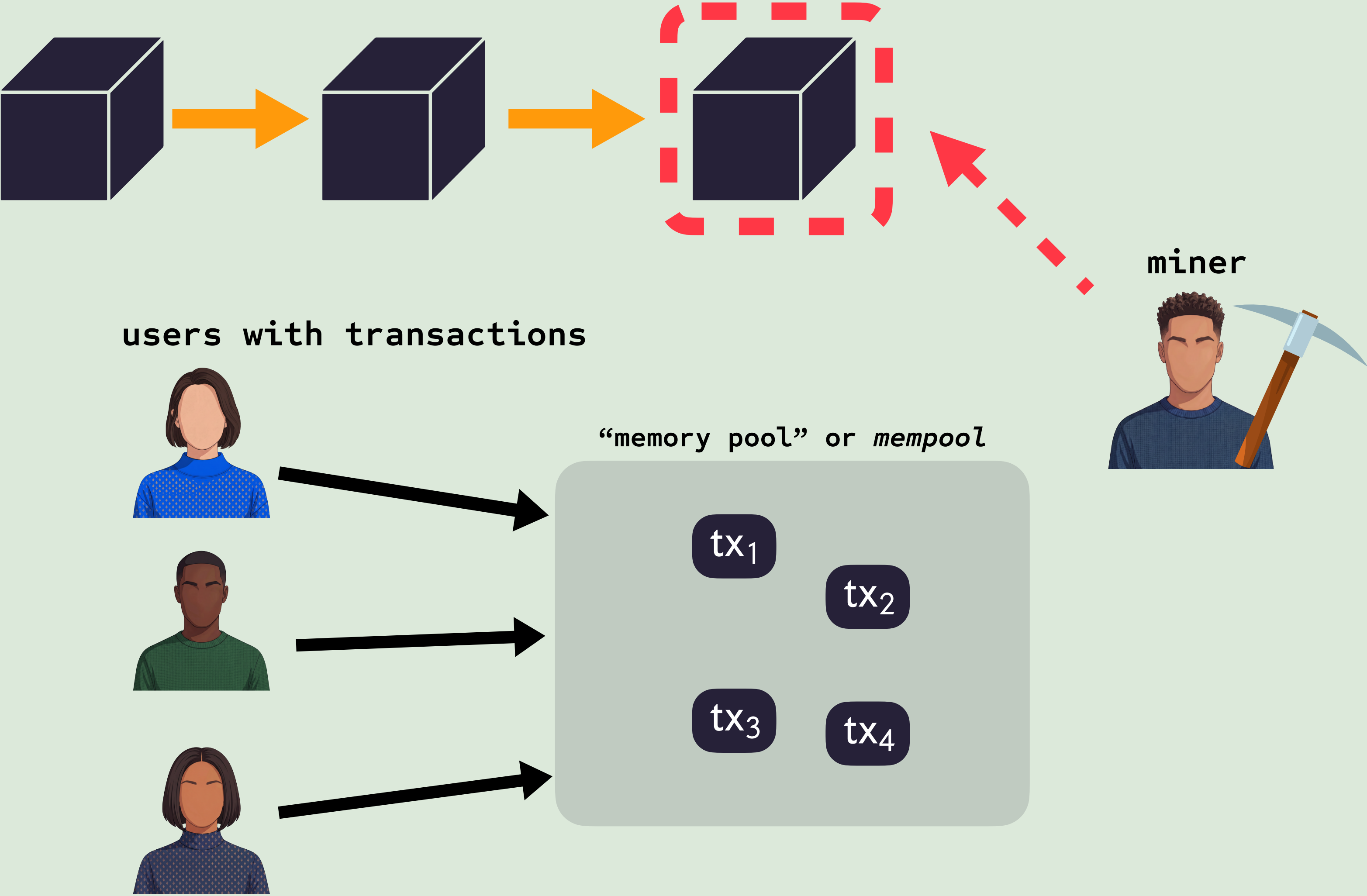
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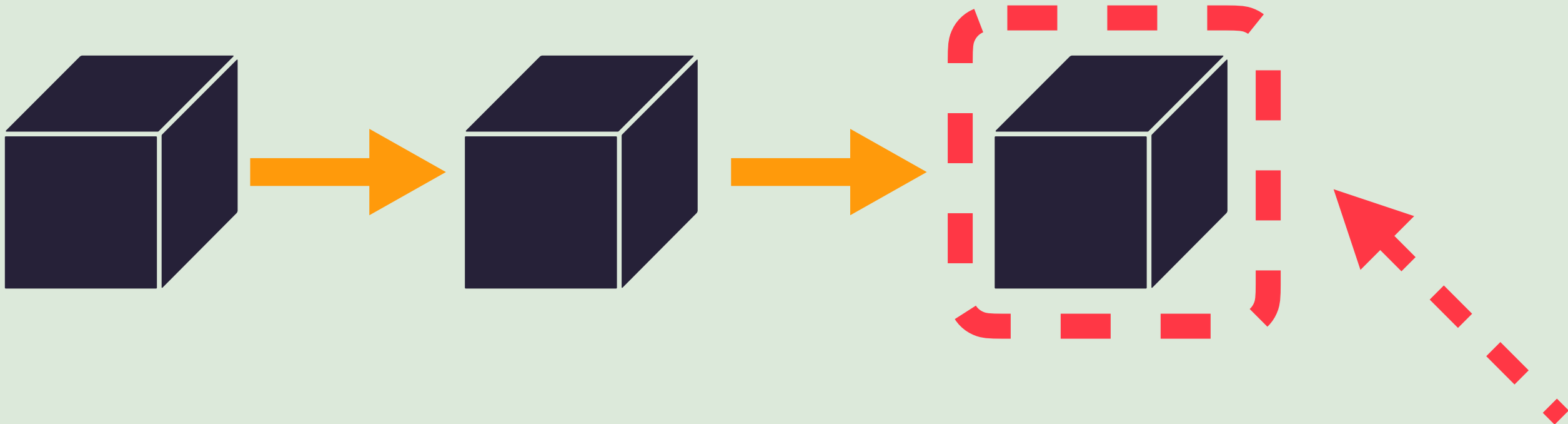
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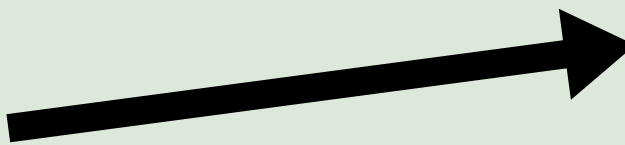
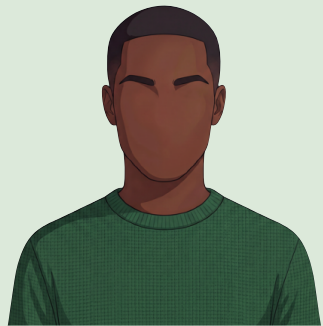
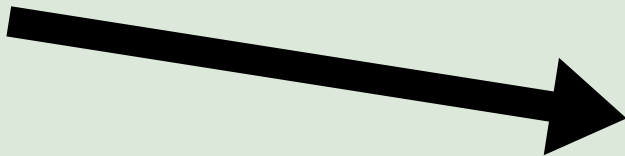


Transactions on block are
executed sequentially.
Order matters!

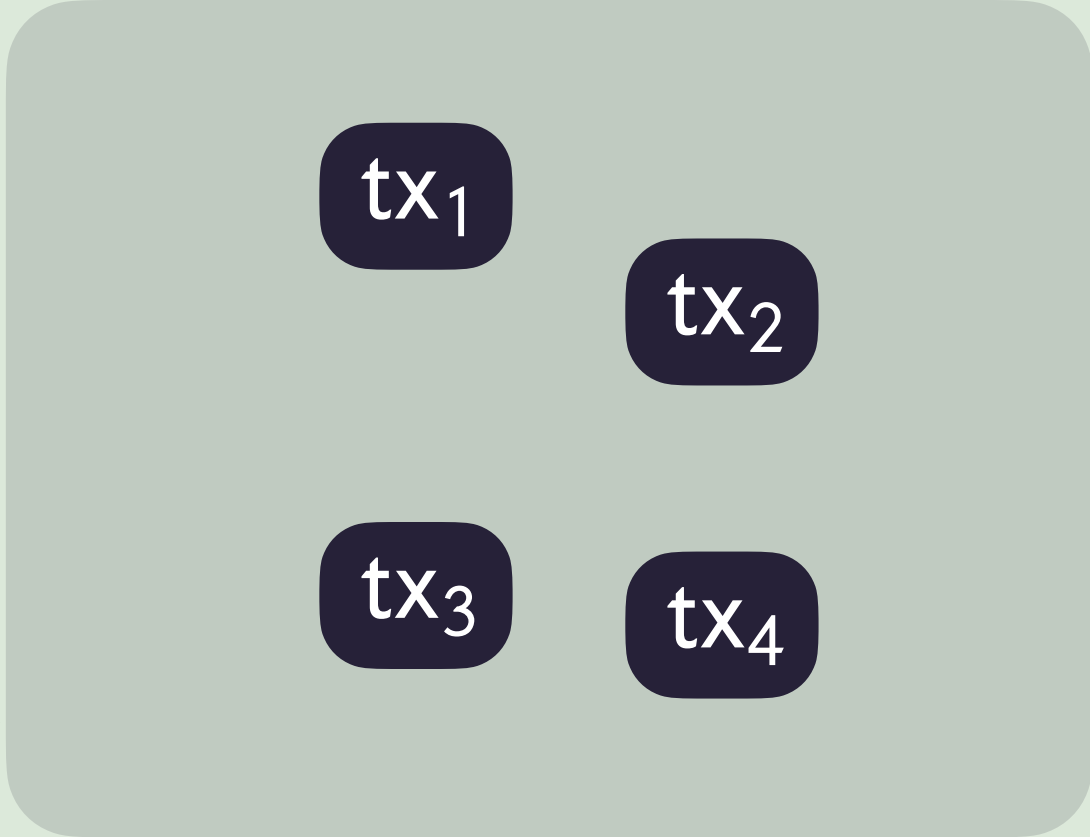


Block proposer can order/
pick in a way that's
beneficial to them (MEV
attacks e.g. front-running)

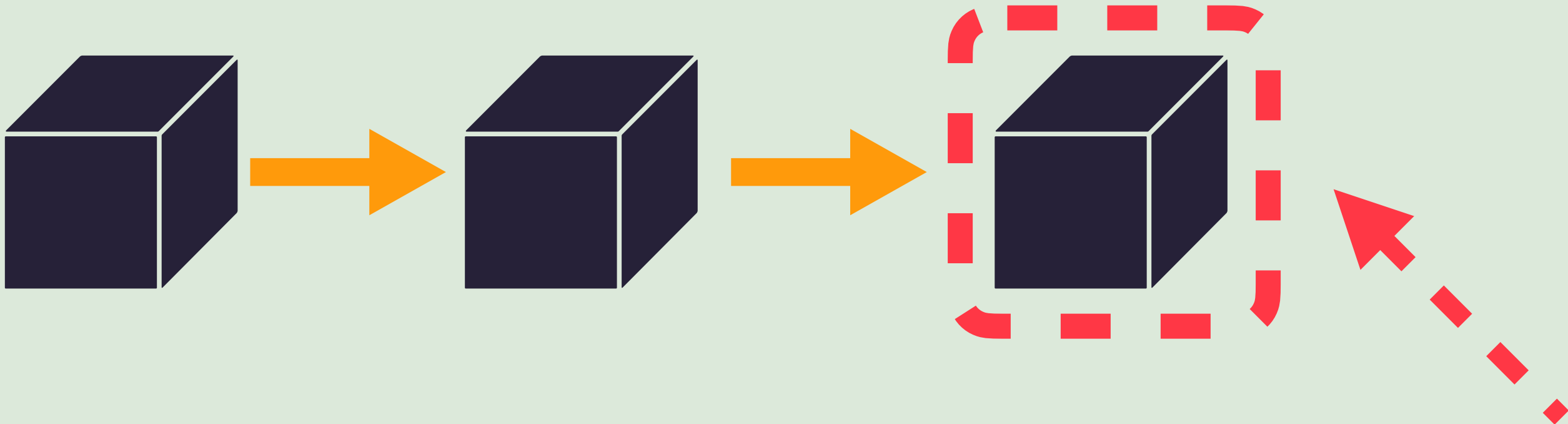
users with transactions



“memory pool” or *mempool*

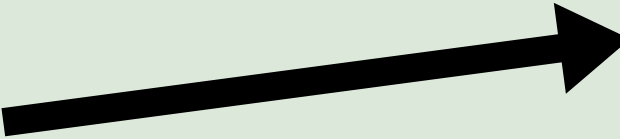
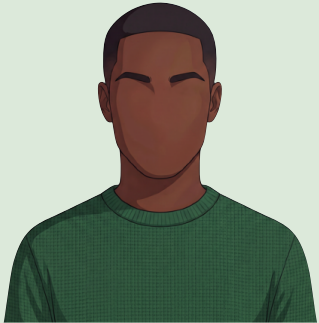
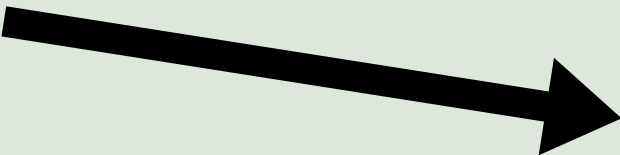


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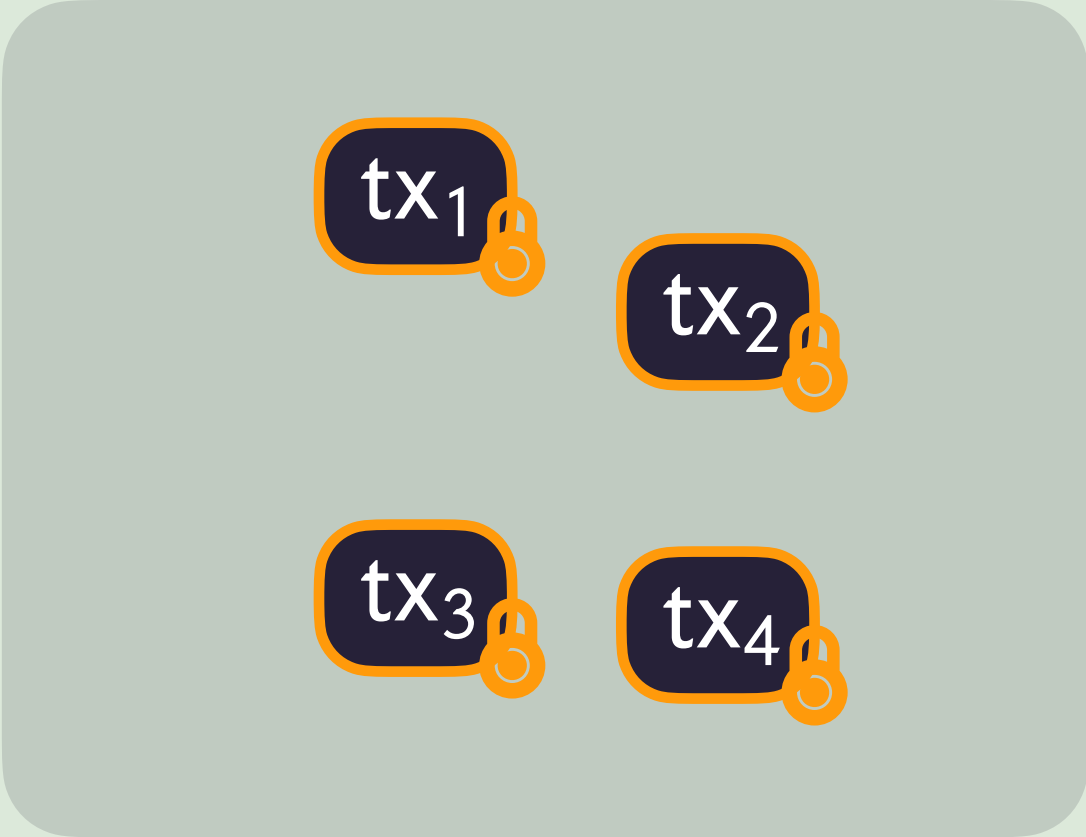


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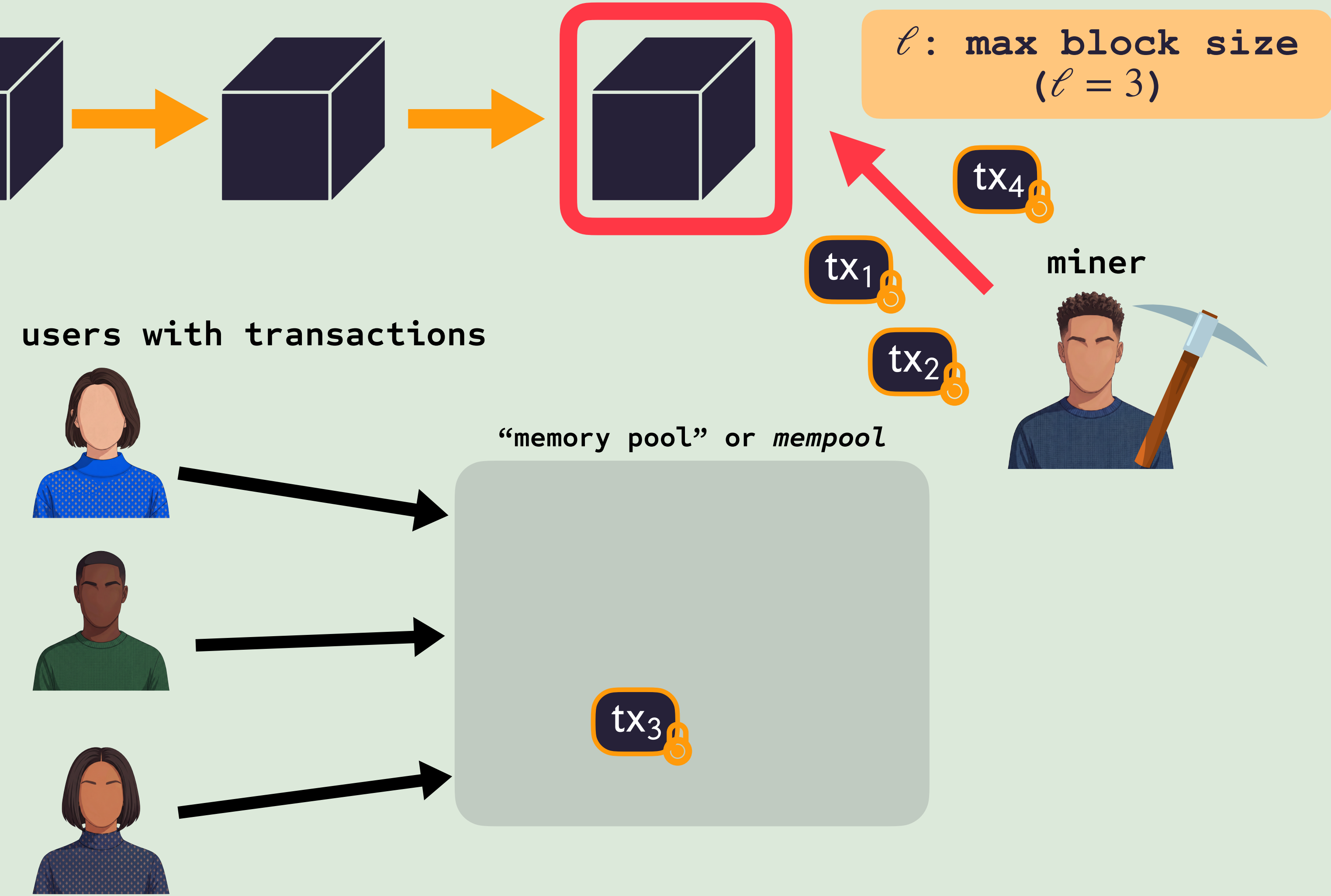
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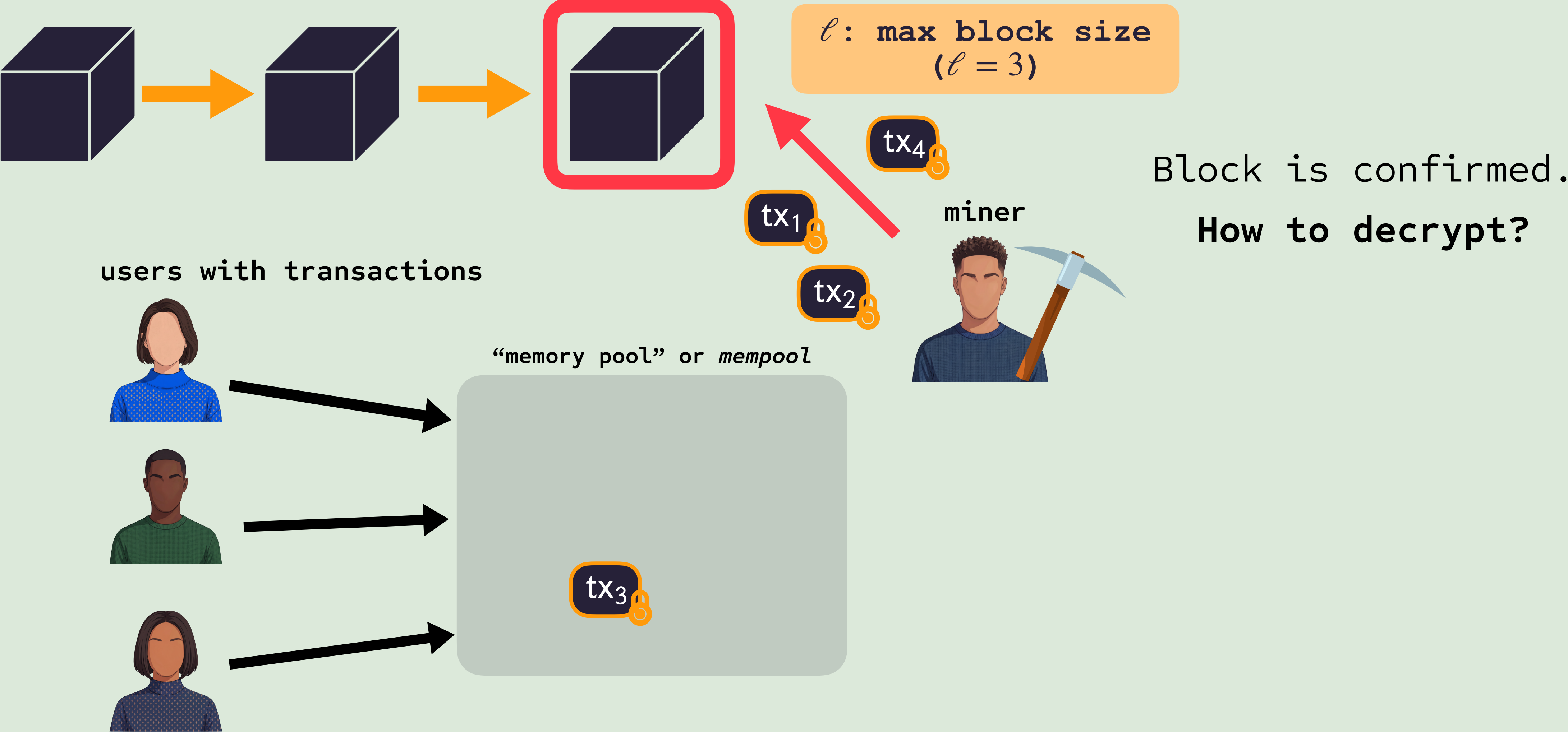
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Can we hide them?
Yes, we could encrypt...

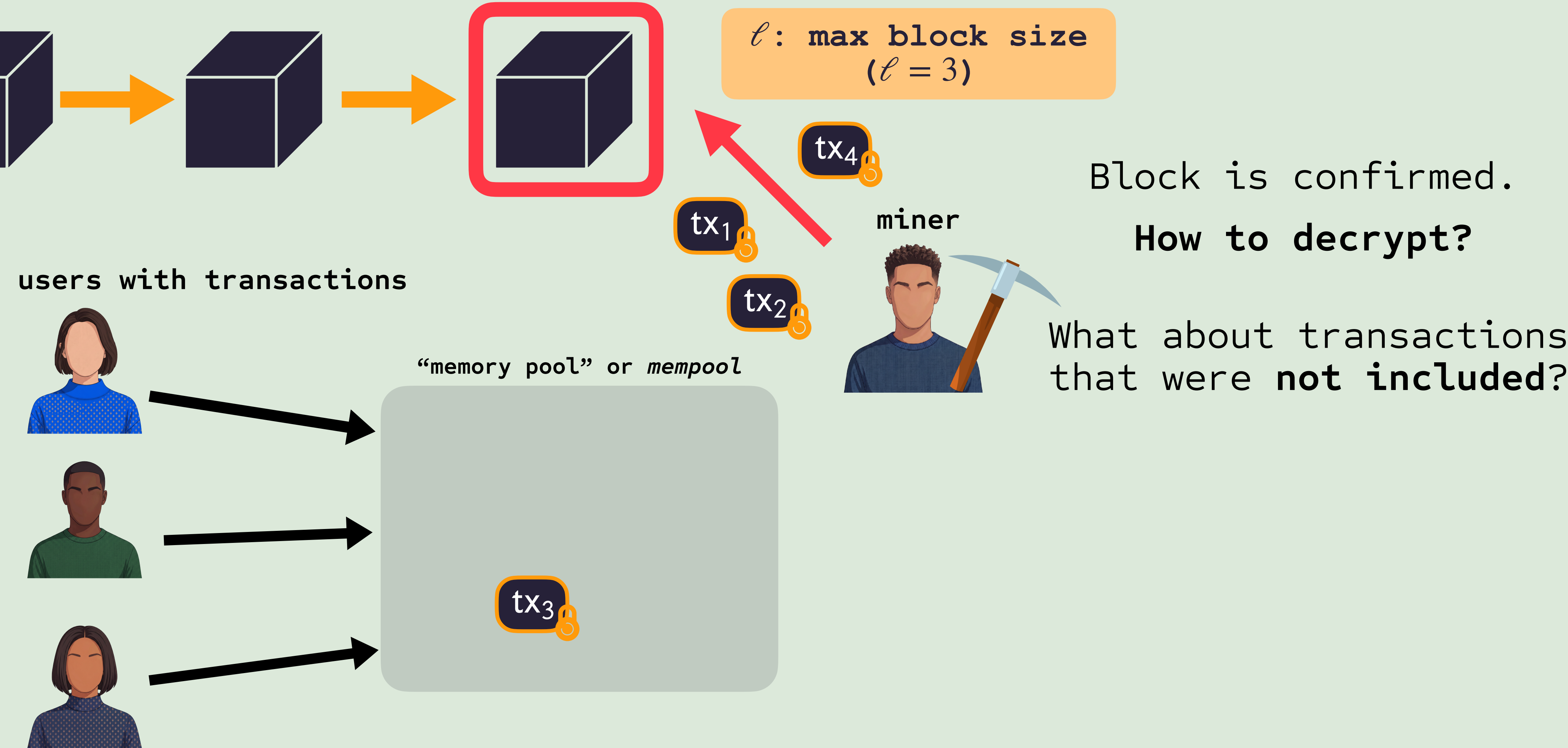
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