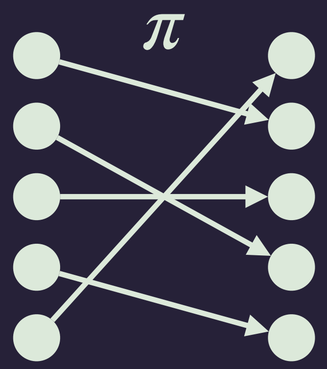
CRYPTO 2026 | AUGUST 19, 2026



Linear-time Shuffling With Malicious Security

begone selective failure attacks!

Rohit Nema (Stanford)

joint work with

Samuel Dittmer
(Stealth Software Technologies)

Rafail Ostrovsky
(UCLA)

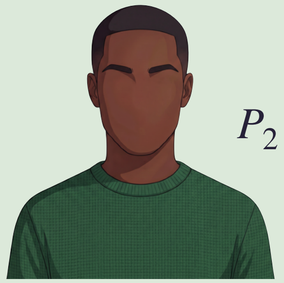
Paper: ia.cr/2025/2137

The Problem: Shuffling Secret-shared Values

P_1

Vector of values $\vec{v}$

v_1 v_2 v_3 v_4 v_5



P_2

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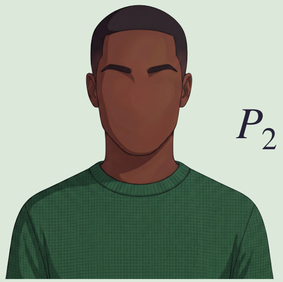

P_2

input

(additive) sharing of $\vec{v}$

$[v_1]_1$ $[v_2]_1$ $[v_3]_1$ $[v_4]_1$ $[v_5]_1$

$[v_1]_2$ $[v_2]_2$ $[v_3]_2$ $[v_4]_2$ $[v_5]_2$

$$[v_i]_1 + [v_i]_2 = v_i$$

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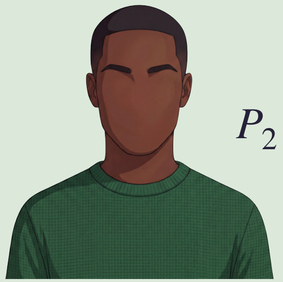

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$$[v_i]_1 + [v_i]_2 = v_i$$

Goal

Obtain fresh shares of

$$\vec{w} = \pi(\vec{v})$$

for *unknown* permutation π

The Problem: Shuffling Secret-shared Values

Vector of values $\vec{v}$

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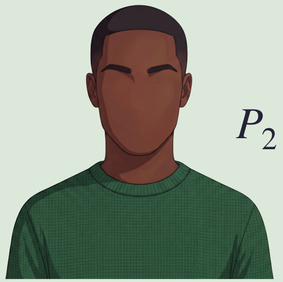

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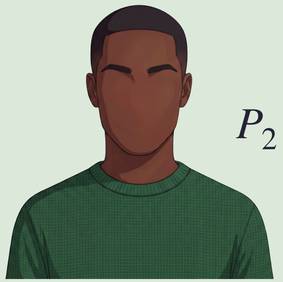

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MPC protocol:

random permutation π

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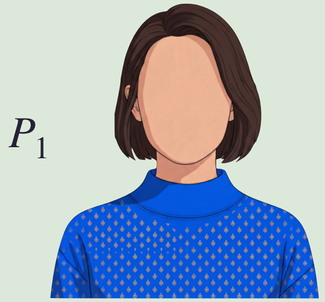

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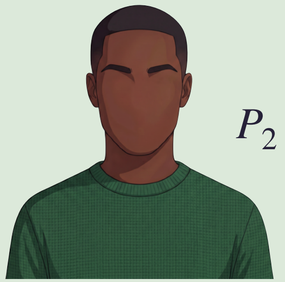

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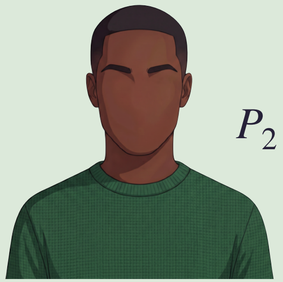

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output

sharing of $\vec{w} = \pi(\vec{v})$

$$[w_i]_1 + [w_i]_2 = \pi(\vec{v})_i$$

Relaxation: “Half” Shuffle

Vector of values $\vec{v}$

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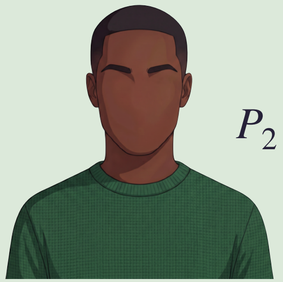

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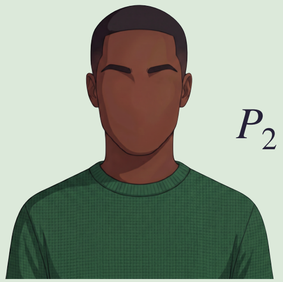

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Picks a permutation π

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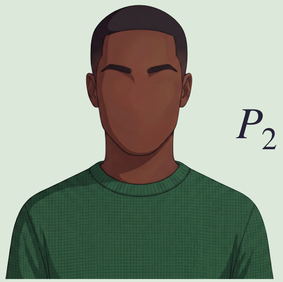

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MPC protocol

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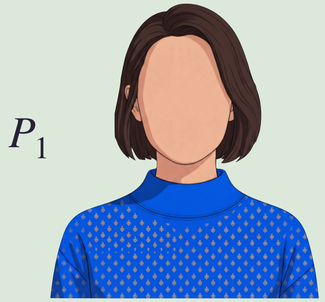

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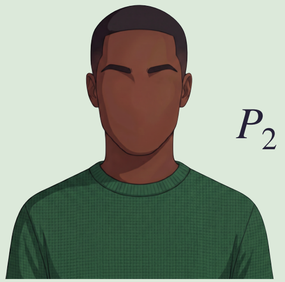

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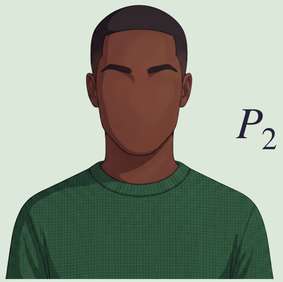

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Picks a permutation π

MPC protocol

output

sharing of $\vec{w} = \pi(\vec{v})$

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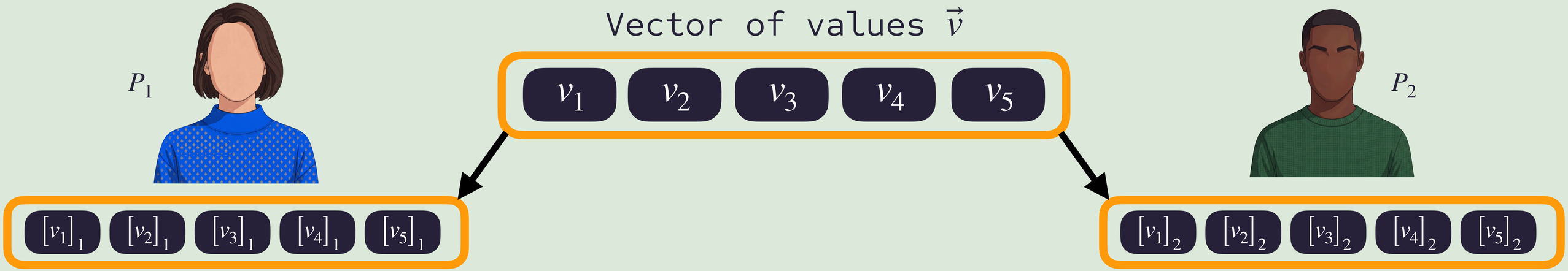
Shuffle = Two Half-Shuffles



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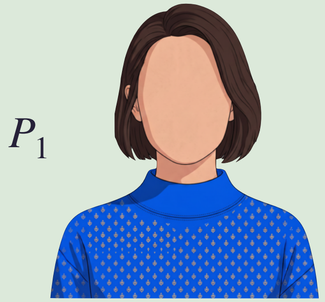

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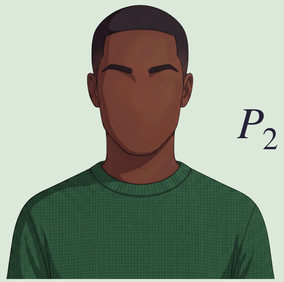

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π_1

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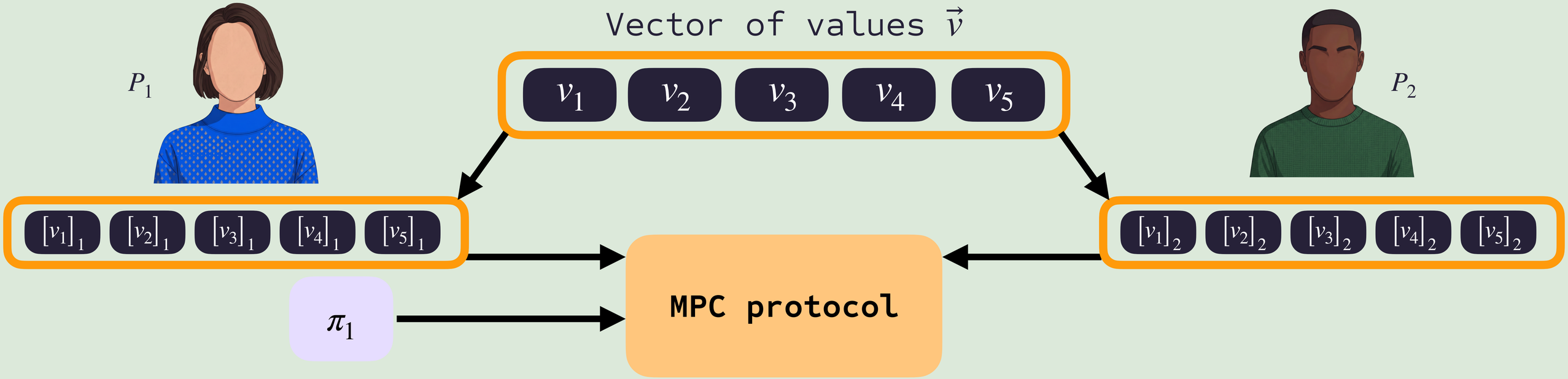

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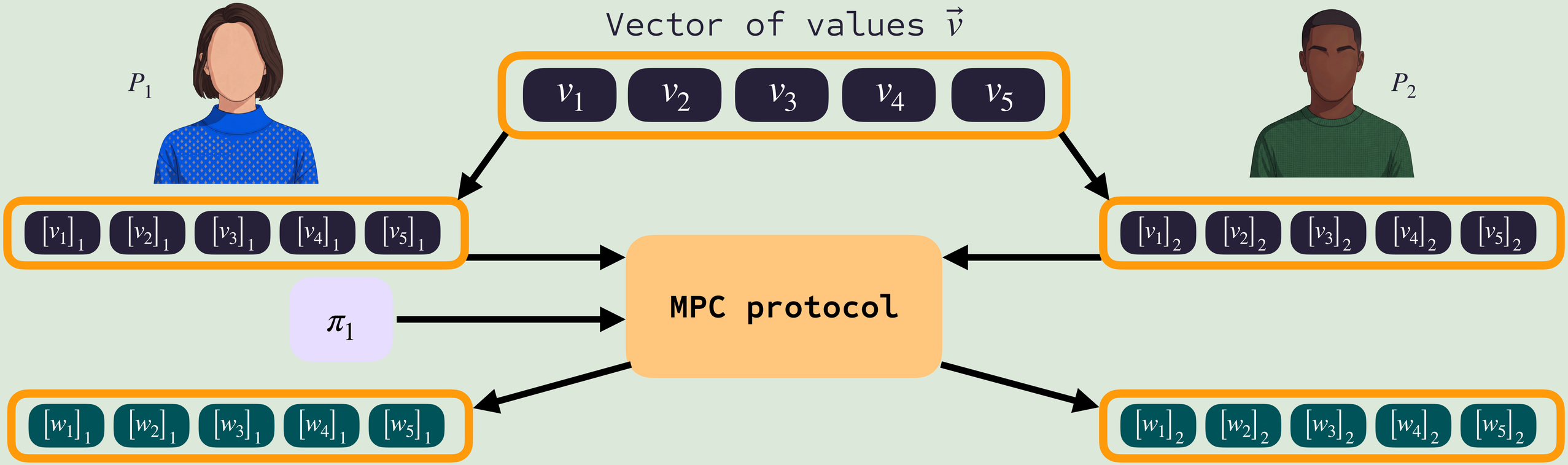

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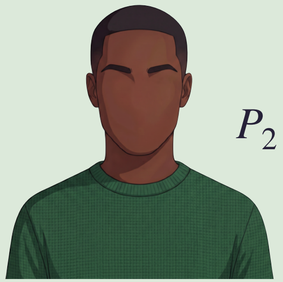

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MPC protocol

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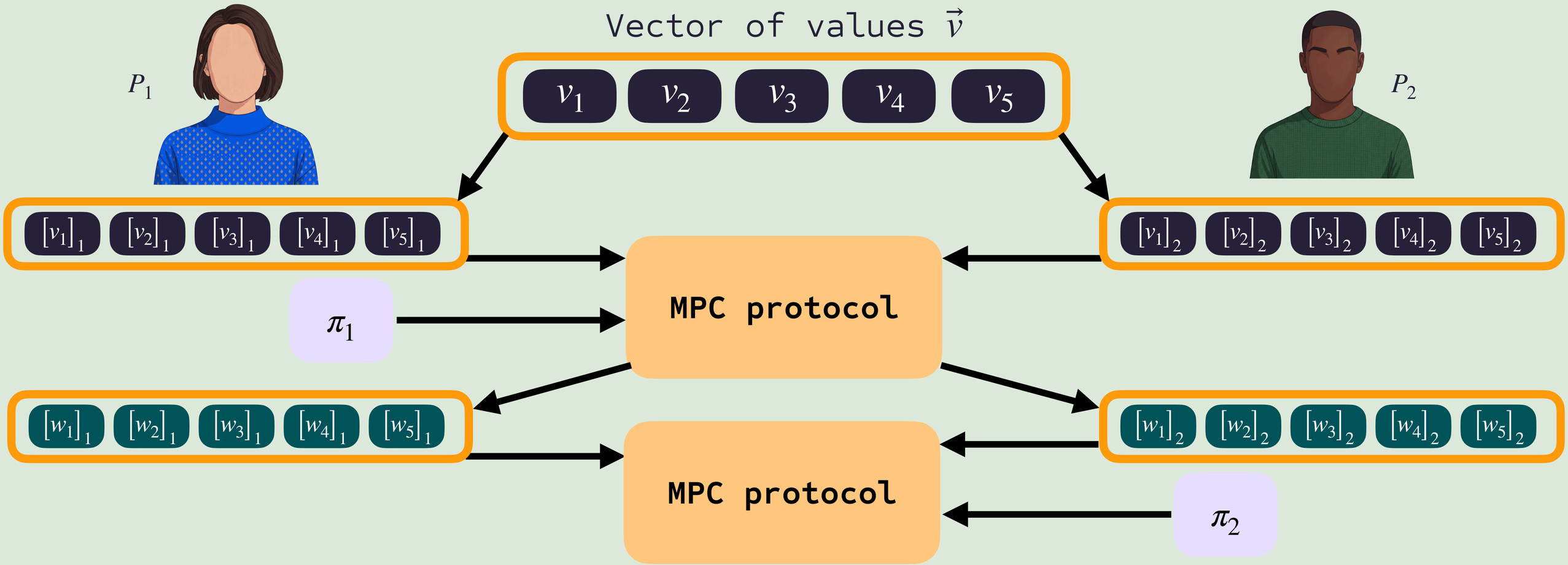

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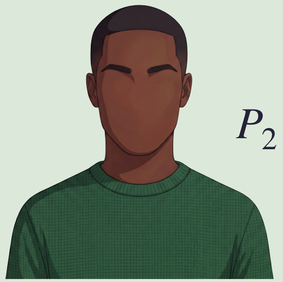

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MPC protocol

π_2

$[z_1]_1$ $[z_2]_1$ $[z_3]_1$ $[z_4]_1$ $[z_5]_1$

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final output

sharing of $\vec{z} = (\pi_2 \circ \pi_1)(\vec{v})$

Shuffle = Two Half-Shuffles

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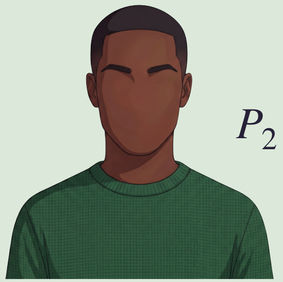

P_2

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π_1

MPC protocol

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MPC protocol

π_2

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final output

sharing of $\vec{z} = (\pi_2 \circ \pi_1)(\vec{v})$

$\pi_2 \circ \pi_1$ appears random to both P_1 and P_2

Oblivious Shuffling

Oblivious Shuffling

Highly active area of research

Loads of work on this in various settings:

Permutation matrix: [CDI+24]

Homormorphic encryption-based (folklore): [HEK12,GHJ+15,FO21]

Offline-online: [CGP20,Lau21,EB22,SYB+24,CNO+26]

Verifiable computation: [CF13, GW13, LWZ18, FNP20, BCF+21, GNS23, KVM+24, ACG+25, CCC+25, ...]

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either **only semi-honest security** or **super-linear** in offline or online computation/communication

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Applications

Essential building block for privacy-preserving protocols e.g. in

(1) analytics

(2) messaging

(3) RAM computations

and many more...

Our Result

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Asymptotically linear

Linear-time and space protocol by dealing with the RAM computation directly instead of compiling a circuit

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Malicious security

We obtain **security *with abort*** against a **malicious** adversary

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Linear-time and space protocol by dealing with the RAM computation directly instead of compiling a circuit

Malicious security

We obtain **security *with abort*** against a **malicious** adversary

A new assumption

We introduce a **novel weakening of the “linear-only” assumption** on encryption schemes that limits operations on ciphertexts to that of “weak predicates”

Our Construction: Roadmap

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Folklore semi-honest protocol using Linearly Homomorphic Encryption (LHE) and authenticated secret sharing

Our Construction: Roadmap

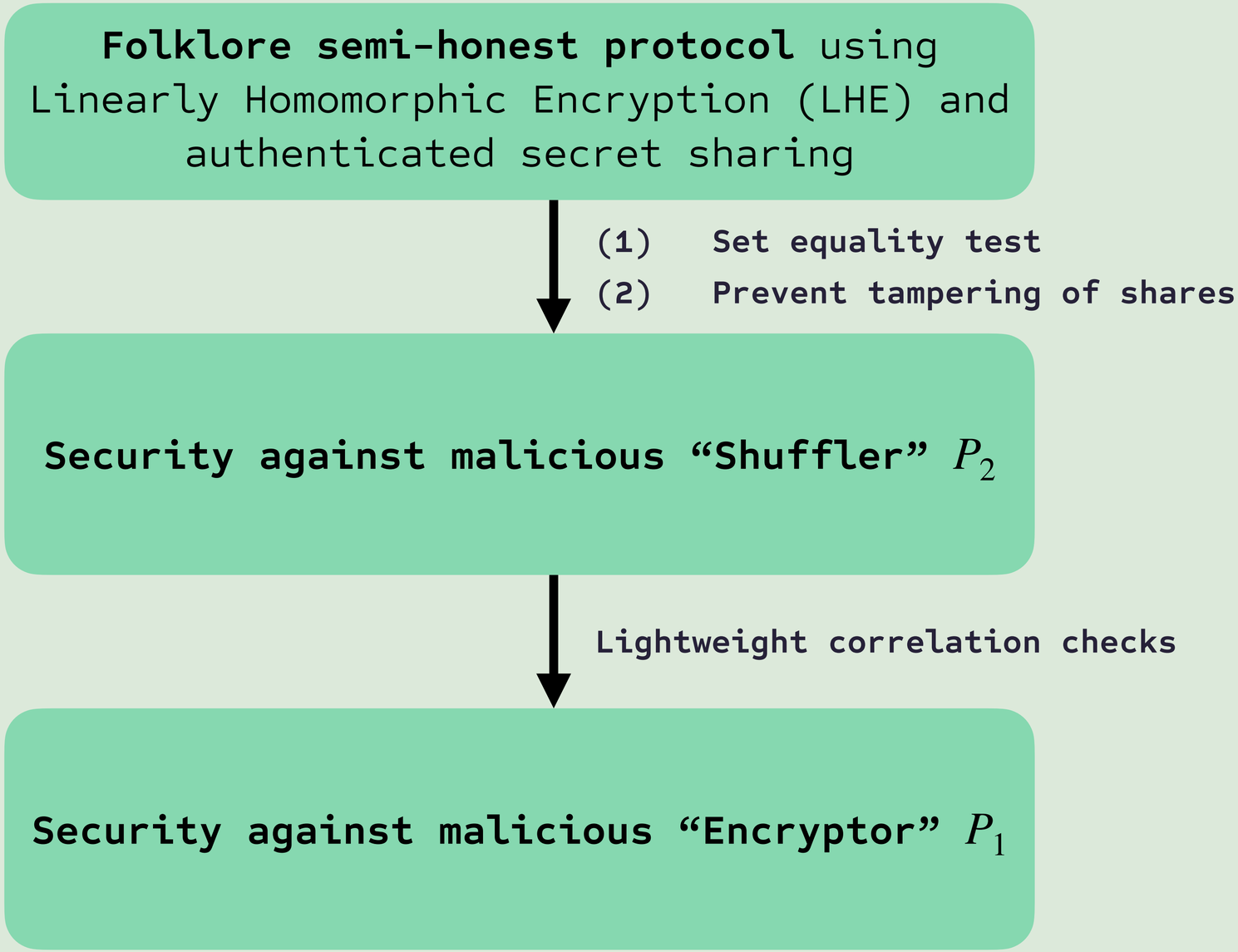
Folklore semi-honest protocol using Linearly Homomorphic Encryption (LHE) and authenticated secret sharing

(1) Set equality test

(2) Prevent tampering of shares

Security against malicious “Shuffler” P_2

Our Construction: Roadmap



Folklore Approach Using LHE

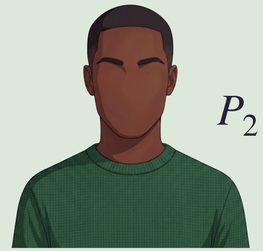
$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

$[\vec{v}]_1$

Encryptor

$[\vec{v}]_2$ π_2



P_2

Shuffler

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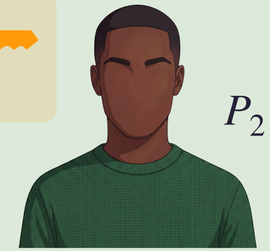

P_2

$[\vec{v}]_2$

π_2

Shuffler

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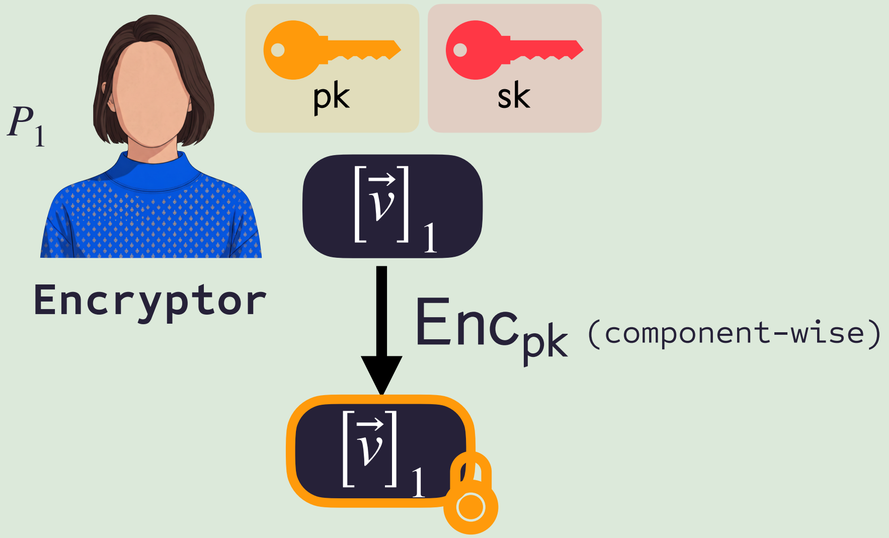

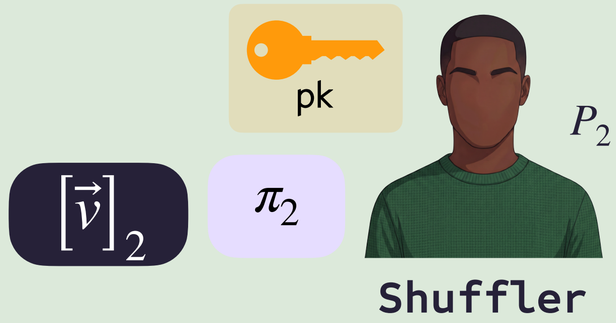

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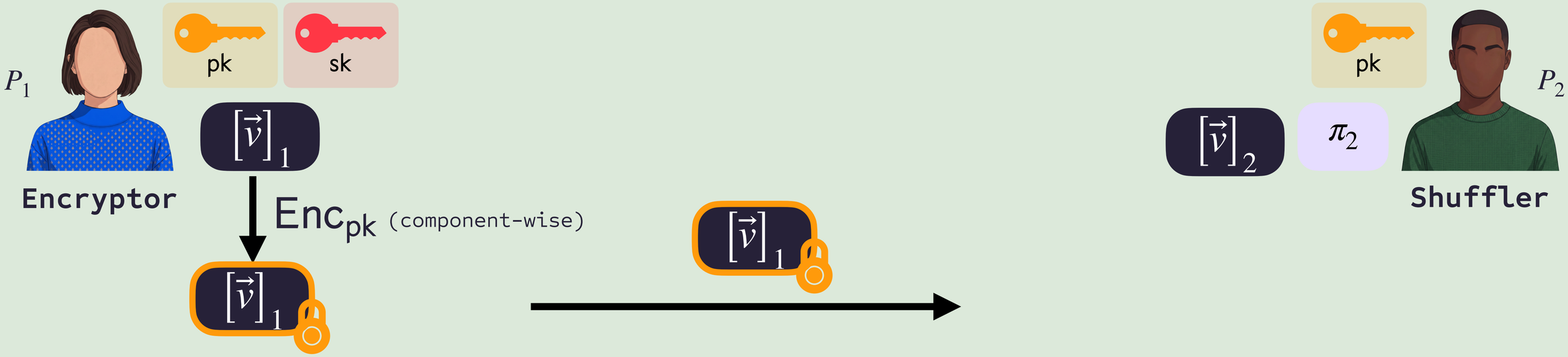

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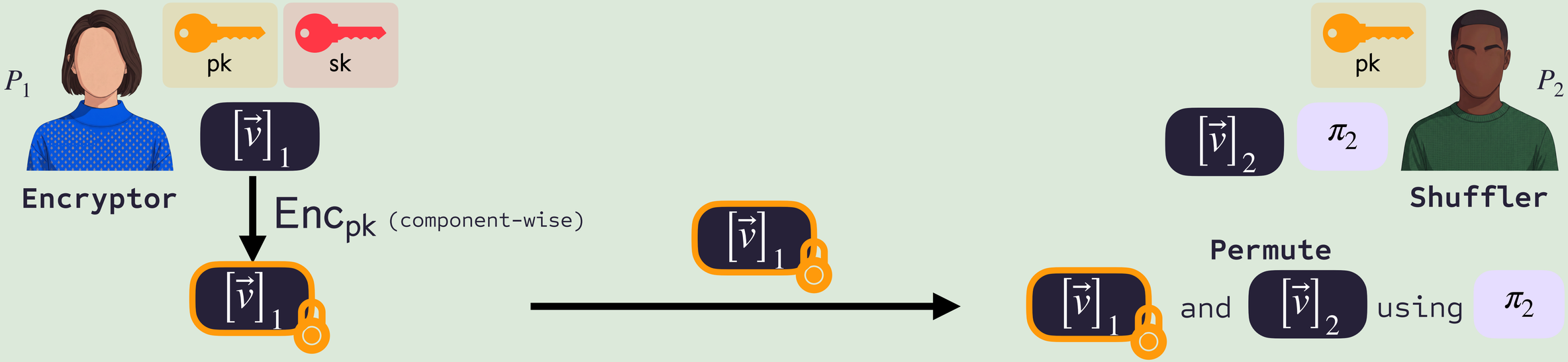
$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$



Folklore Approach Using LHE

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

$[\vec{v}]_1$

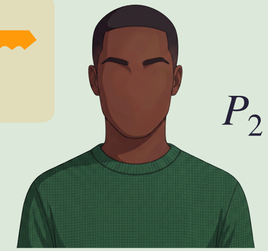
Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

$[\vec{v}]_1$

pk



P_2

$[\vec{v}]_2$

π_2

Shuffler

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$, $\pi([\vec{v}]_2) - \vec{r}$

Folklore Approach Using LHE

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

$[\vec{v}]_1$

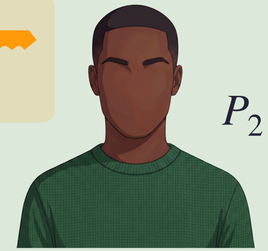
Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

$[\vec{v}]_1$

pk



P_2

$[\vec{v}]_2$

π_2

Shuffler

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_2) - \vec{r}$

Folklore Approach Using LHE

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

$[\vec{v}]_1$

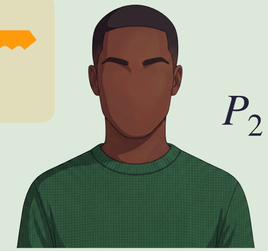
Encryptor

Enc_{pk} (component-wise)

$[\vec{v}]_1$

$[\vec{v}]_1$

pk



P_2

$[\vec{v}]_2$

π_2

Shuffler

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$, $\pi([\vec{v}]_2) - \vec{r}$

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r} = \pi([\vec{v}]_1) + \vec{r}$

Folklore Approach Using LHE

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

$[\vec{v}]_1$

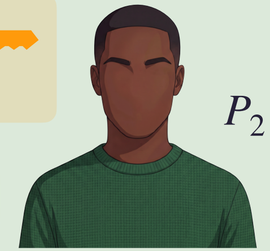
Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

$[\vec{v}]_1$

pk



P_2

$[\vec{v}]_2$

π_2

Shuffler

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$, $\pi([\vec{v}]_2) - \vec{r}$

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r} = \pi([\vec{v}]_1) + \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

Folklore Approach Using LHE

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

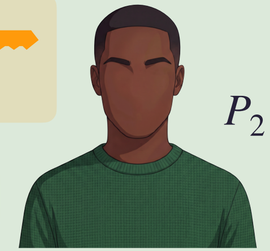
$[\vec{v}]_1$

Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

$[\vec{v}]_1$



P_2

pk

$[\vec{v}]_2$

π_2

Shuffler

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$, $\pi([\vec{v}]_2) - \vec{r}$

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r} = \pi([\vec{v}]_1) + \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$\pi([\vec{v}]_1) + \vec{r}$

Folklore Approach Using LHE

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

$[\vec{v}]_1$

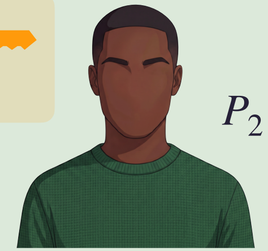
Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

$[\vec{v}]_1$

pk



P_2

$[\vec{v}]_2$

π_2

Shuffler

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$, $\pi([\vec{v}]_2) - \vec{r}$

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r} = \pi([\vec{v}]_1) + \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_2) - \vec{r}$

Folklore Approach Using LHE

$[\vec{v}]_i = [[v_1]_i, [v_2]_i, [v_3]_i, [v_4]_i, [v_5]_i]$

P_1

pk

sk

Encryptor

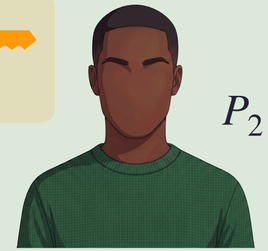
$[\vec{v}]_1$

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

pk

P_2



$[\vec{v}]_2$

π_2

Shuffler

$[\vec{v}]_1$

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$, $\pi([\vec{v}]_2) - \vec{r}$

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r} = \pi([\vec{v}]_1) + \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$[\vec{w}]_1 = \pi([\vec{v}]_1) + \vec{r}$

$[\vec{w}]_2 = \pi([\vec{v}]_2) - \vec{r}$

set output shares

Folklore Approach Using LHE

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

Encryptor

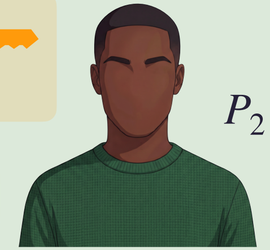
$[\vec{v}]_1$

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

$[\vec{v}]_1$

pk



P_2

$[\vec{v}]_2$

π_2

Shuffler

Permute

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

Linearly re-randomize

$\pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r} = \pi([\vec{v}]_1) + \vec{r}$

$\pi([\vec{v}]_1) + \vec{r}$, $\pi([\vec{v}]_2) - \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$[\vec{w}]_1 = \pi([\vec{v}]_1) + \vec{r}$

$[\vec{w}]_2 = \pi([\vec{v}]_2) - \vec{r}$

concretely expensive due to public key operations but asymptomatically linear nonetheless...

Not maliciously secure...

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

$[\vec{v}]_1$

Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

pk

P_2

$[\vec{v}]_2$

π_2

Shuffler

$[\vec{v}]_1$

$[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

can modify shares arbitrarily

$\pi\left([\vec{v}]_1\right) + \vec{r}$

$\pi\left([\vec{v}]_1\right) + \vec{r} = \pi\left([\vec{v}]_1 + \vec{r}\right)$

$\left([\vec{v}]_2\right) - \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$[\vec{w}]_1 = \pi\left([\vec{v}]_1\right) + \vec{r}$

set output shares

$[\vec{w}]_2 = \pi\left([\vec{v}]_2\right) - \vec{r}$

Not maliciously secure...

$[\vec{v}]_i = [v_1]_i \; [v_2]_i \; [v_3]_i \; [v_4]_i \; [v_5]_i$

P_1

pk

sk

$[\vec{v}]_1$

Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$[\vec{v}]_1$

pk

P_2

$[\vec{v}]_2$

π_2

Shuffler

$[\vec{v}]_1$

Permute $[\vec{v}]_1$ and $[\vec{v}]_2$ using π_2

$\pi([\vec{v}]_1) + \vec{r}$

can modify shares arbitrarily

$\pi([\vec{v}]_1) + \vec{r} = \pi([\vec{v}]_1) + \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$[\vec{w}]_1 = \pi([\vec{v}]_1) + \vec{r}$

set output sh

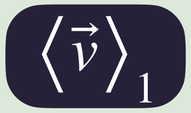
Reconstruct $([\vec{v}]_2) - \vec{r}$

Solution?

use *authenticated* secret sharing

With Authentication

P_1

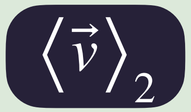


Encryptor

e.g. SPDZ: $\langle x \rangle = ([x], [\alpha \cdot x])$

for $\alpha \xleftarrow{\$} \mathbb{F}$

P_2



Shuffler

With Authentication

P_1

pk

sk

Encryptor

e.g. SPDZ: $\langle x \rangle = ([x], [\alpha \cdot x])$ for $\alpha \xleftarrow{\$} \mathbb{F}$

pk

P_2

Shuffler

$\langle \vec{v} \rangle_2$ π_2

$\langle \vec{v} \rangle_1$

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$\langle \vec{v} \rangle_1$

$\langle \vec{v} \rangle_1$

Permute $\langle \vec{v} \rangle_1$ and $\langle \vec{v} \rangle_2$ using π_2

Linearly re-randomize

$\pi(\langle \vec{v} \rangle_1) + \vec{r}$, $\pi(\langle \vec{v} \rangle_2) - \vec{r}$

$\pi(\langle \vec{v} \rangle_1) + \vec{r}$

$\pi(\langle \vec{v} \rangle_1) + \vec{r} = \pi(\langle \vec{v} \rangle_1) + \vec{r}$

(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$\langle \vec{w} \rangle_1 = \pi(\langle \vec{v} \rangle_1) + \vec{r}$

set output shares

$\langle \vec{w} \rangle_2 = \pi(\langle \vec{v} \rangle_2) - \vec{r}$

With Authentication

P_1

pk

sk

$\langle \vec{v} \rangle_1$

Encryptor

e.g. SPDZ: $\langle x \rangle = ([x], [\alpha \cdot x])$ for $\alpha \xleftarrow{\$} \mathbb{F}$

pk

P_2

$\langle \vec{v} \rangle_2$

π_2

Shuffler

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$\langle \vec{v} \rangle_1$

$\langle \vec{v} \rangle_1$

$\langle \vec{v} \rangle_1$ and $\langle \vec{v} \rangle_2$ using π_2

can no longer modify arbitrarily without being caught by authentication check

$\pi(\langle \vec{v} \rangle_1) + \vec{r}$

$\pi(\langle \vec{v} \rangle_1) + \vec{r} = \pi(\langle \vec{v} \rangle_1 + \vec{r})$

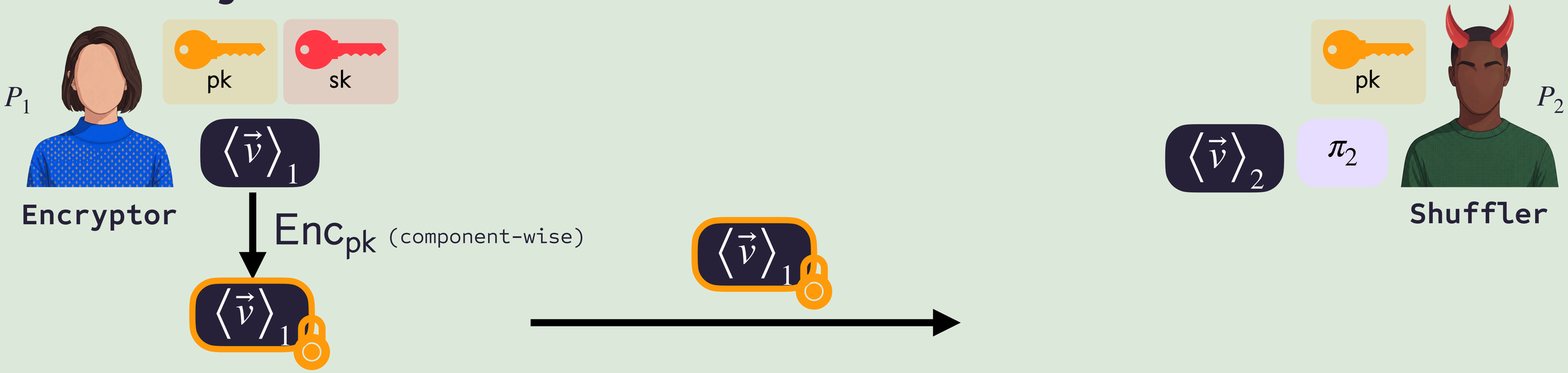
(component-wise) $\mathsf{Dec}_{\mathsf{sk}}$

$\langle \vec{w} \rangle_1 = \pi(\langle \vec{v} \rangle_1) + \vec{r}$

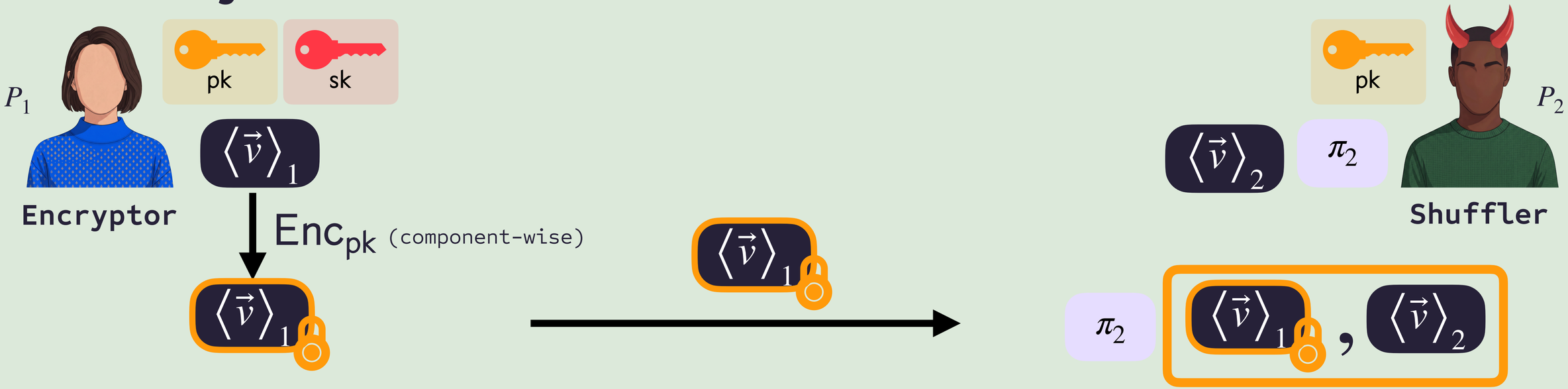
set output shares

$\langle \vec{w} \rangle_2 = \pi(\langle \vec{v} \rangle_2) - \vec{r}$

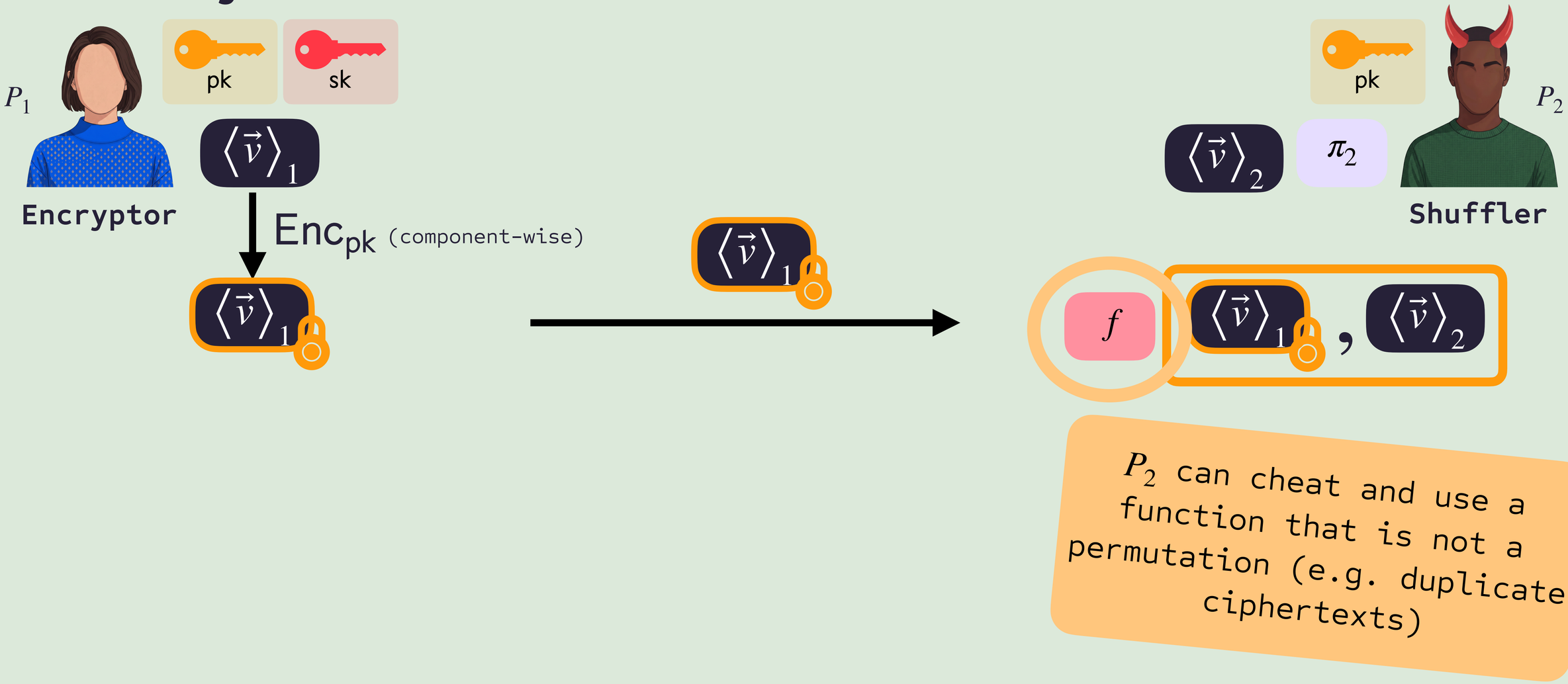
Validity of Permutation



Validity of Permutation

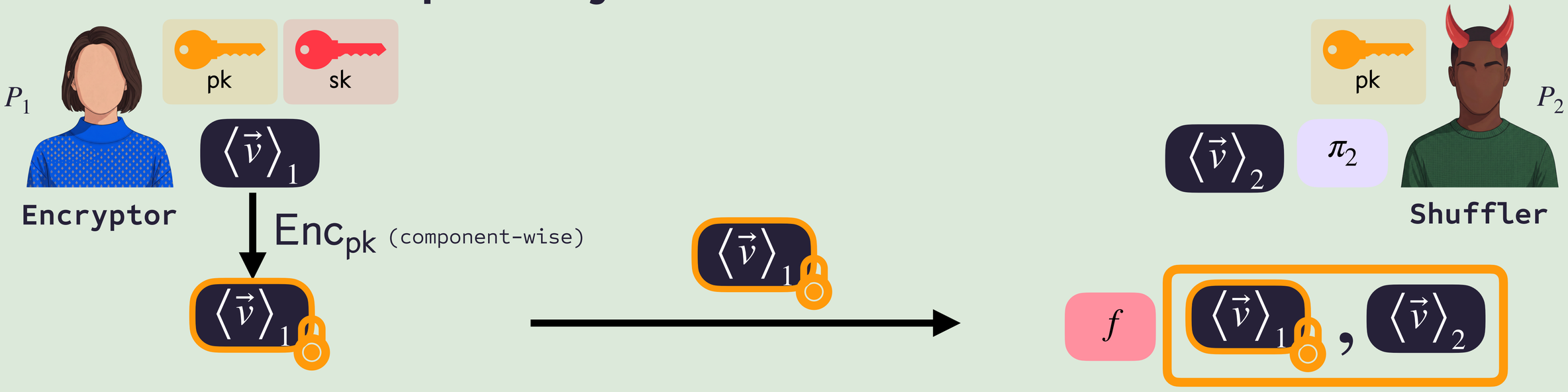


Validity of Permutation



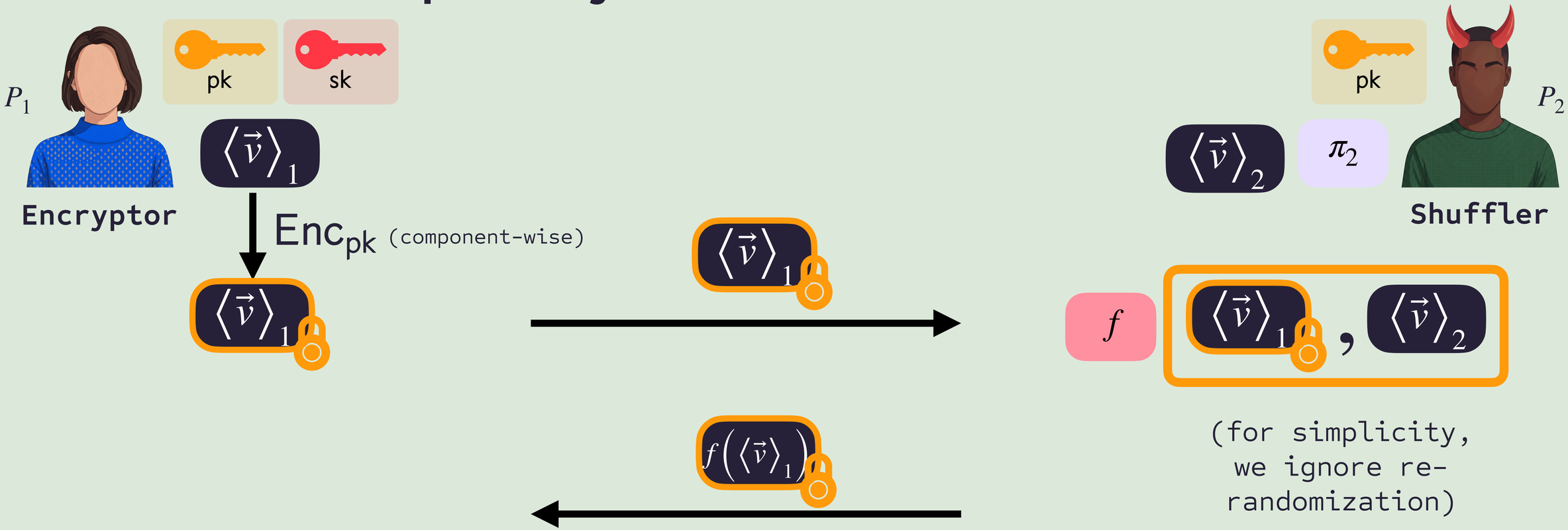
P_2 can cheat and use a function that is not a permutation (e.g. duplicate ciphertexts)

Idea 1: Set Equality Test



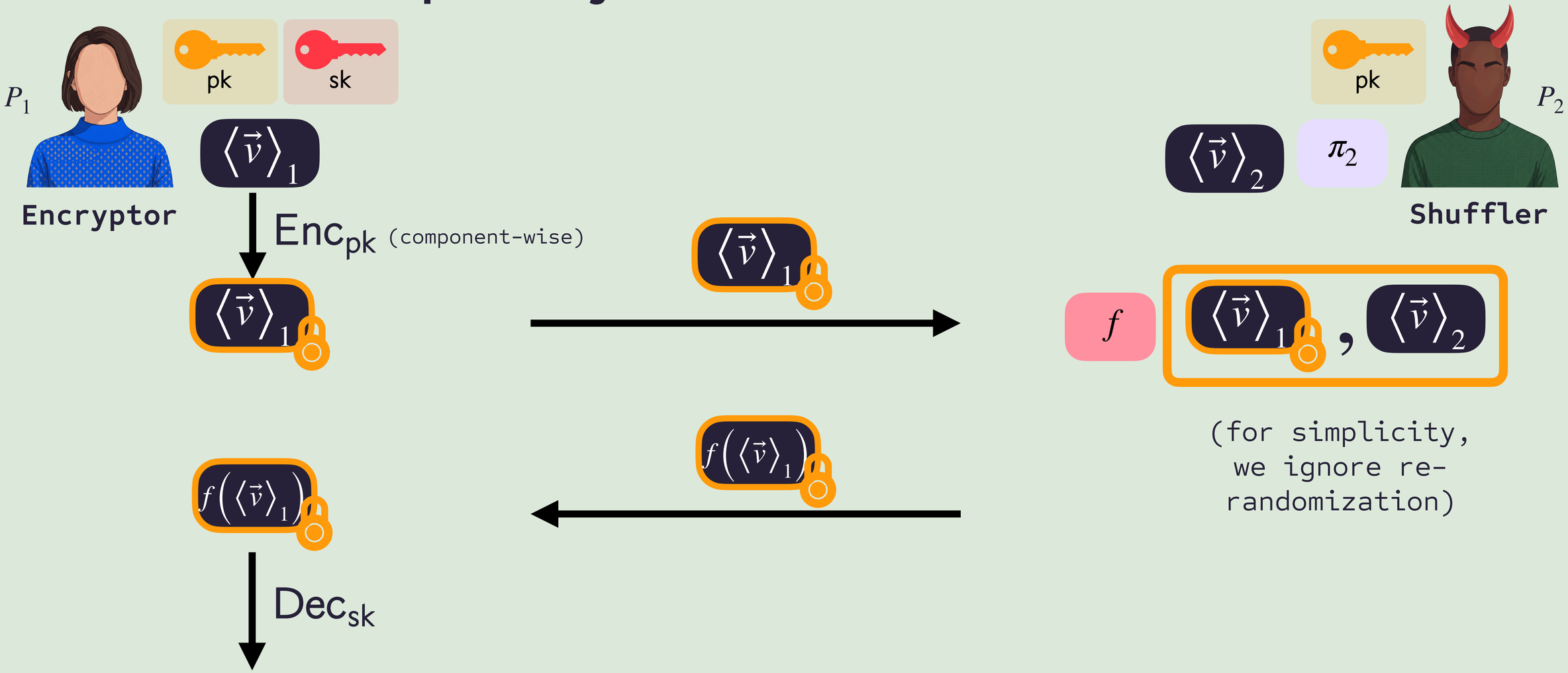
(for simplicity, we ignore re-randomization)

Idea 1: Set Equality Test



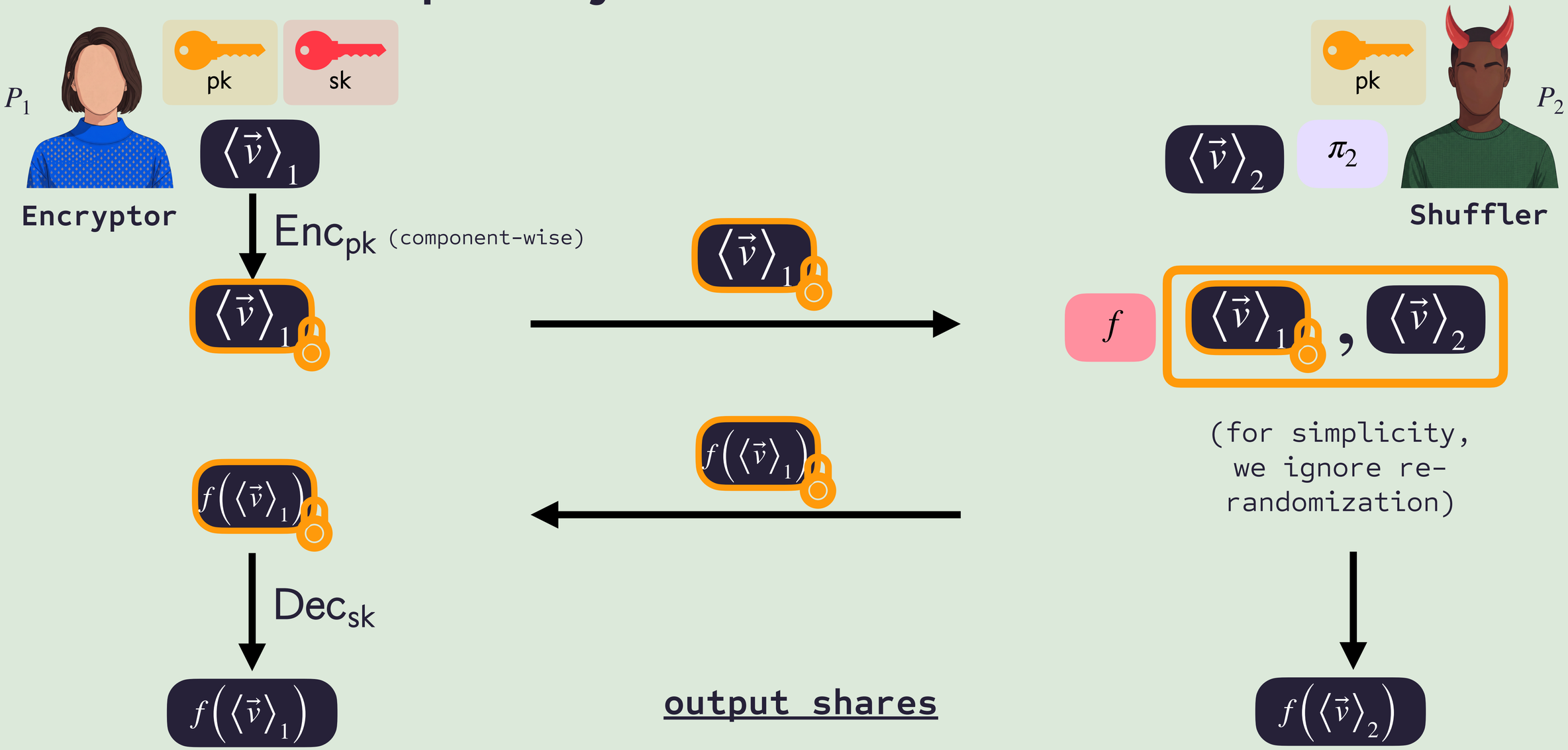
(for simplicity, we ignore re-randomization)

Idea 1: Set Equality Test



(for simplicity, we ignore re-randomization)

Idea 1: Set Equality Test



Idea 1: Set Equality Test

P_1

pk

sk

$\langle \vec{v} \rangle_1$

Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (component-wise)

$\langle \vec{v} \rangle_1$

$\langle \vec{v} \rangle_1$

$\langle \vec{v} \rangle_2$

π_2

pk

P_2

Shuffler

f

$\langle \vec{v} \rangle_1$, $\langle \vec{v} \rangle_2$

(for simplicity, we ignore re-randomization)

Use a set equality test to check the **output shares** represent the **same set** as the **input shares** $\Longrightarrow$ **valid permutation**

$f\left(\langle \vec{v} \rangle_1\right)$

$\mathsf{Dec}_{\mathsf{sk}}$

$f\left(\langle \vec{v} \rangle_1\right)$

output shares

$f\left(\langle \vec{v} \rangle_2\right)$

Idea 1: Set Equality Test

P_1

pk

sk

$\langle \vec{v} \rangle_1$

Encryptor

$\mathsf{Enc}_{\mathsf{pk}}$ (comp

$\langle \vec{v} \rangle_1$

$\langle \vec{v} \rangle_2$

π_2

pk

P_2

Shuffler

BUT... P_2 can now learn information about $\vec{v}$ based on whether the test fails!

$\langle \vec{v} \rangle_1$, $\langle \vec{v} \rangle_2$

Use a set equality test to check the **output shares** represent the **same set** as the **input shares** $\Longrightarrow$ **valid permutation**

(for simplicity, we ignore re-randomization)

$f(\langle \vec{v} \rangle_1)$

$\mathsf{Dec}_{\mathsf{sk}}$

$f(\langle \vec{v} \rangle_1)$

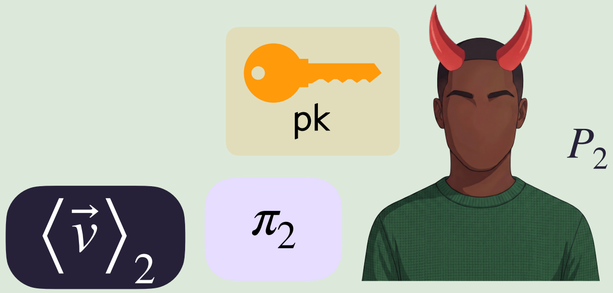
output shares

$f(\langle \vec{v} \rangle_2)$

Selective Failure Attack

Problem:

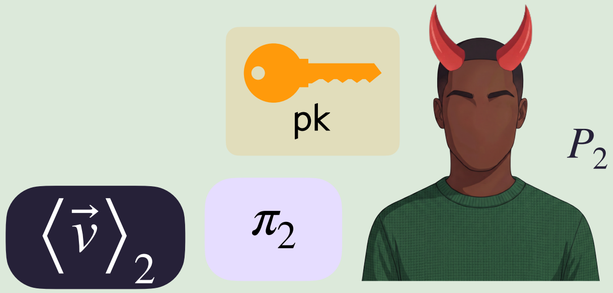
P_2 can exploit linearity to test linear relations over $\vec{v}$



Selective Failure Attack

Problem:

P_2 can exploit linearity to test linear relations over $\vec{v}$



e.g.

test $v_1 + v_2 \stackrel{?}{=} v_3$

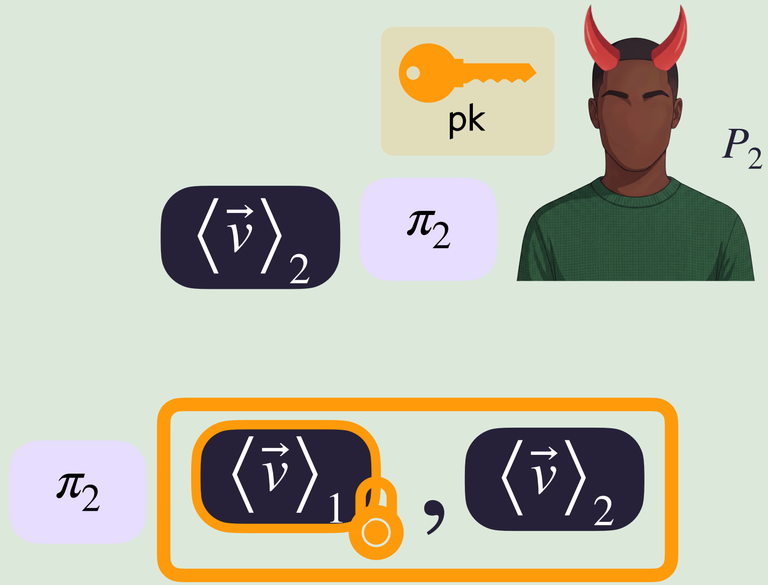
Selective Failure Attack

Problem:

P_2 can exploit linearity to test linear relations over $\vec{v}$

e.g.

test $v_1 + v_2 \stackrel{?}{=} v_3$



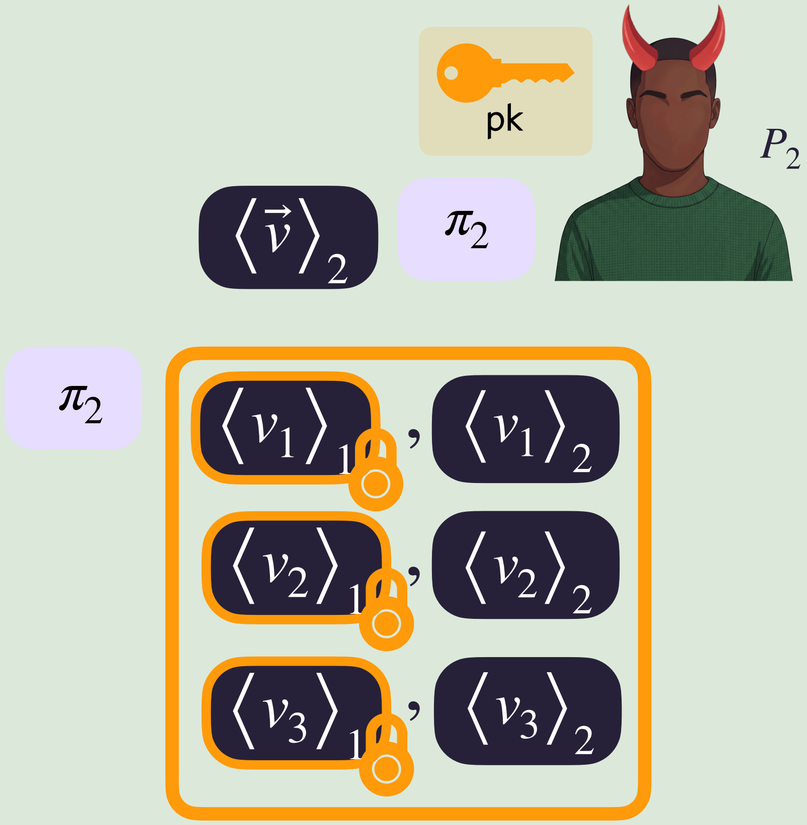
Selective Failure Attack

Problem:

P_2 can exploit linearity to test linear relations over $\vec{v}$

e.g.

test $v_1 + v_2 \stackrel{?}{=} v_3$



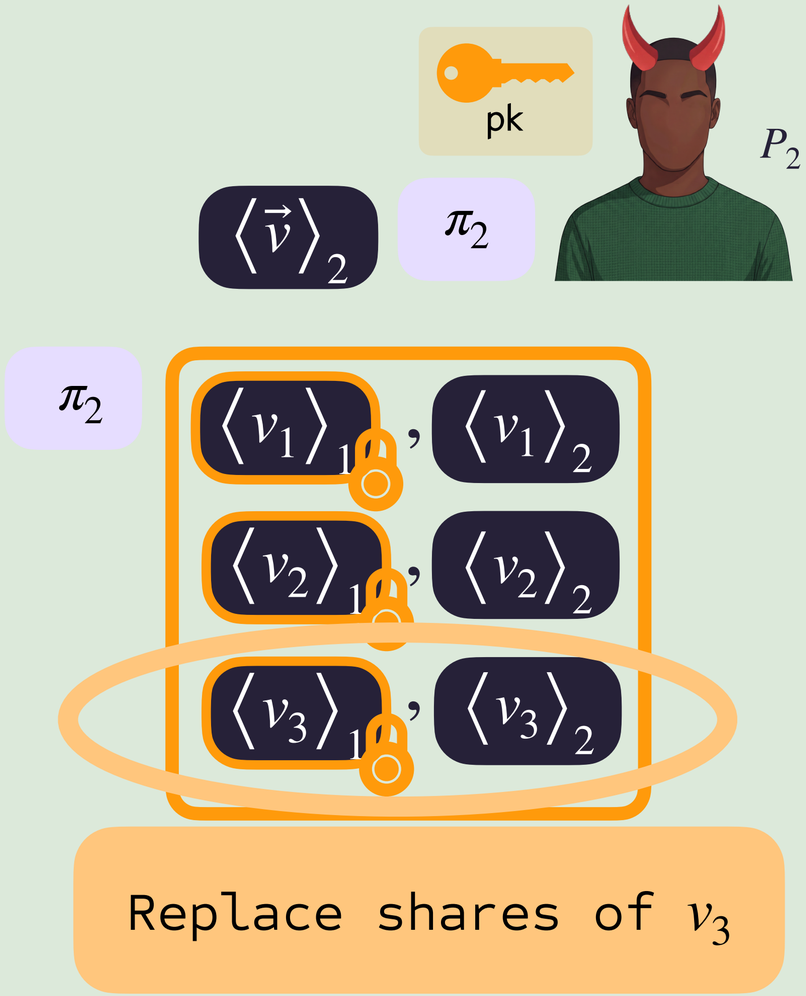
Selective Failure Attack

Problem:

P_2 can exploit linearity to test linear relations over $\vec{v}$

e.g.

test $v_1 + v_2 \stackrel{?}{=} v_3$



Replace shares of v_3

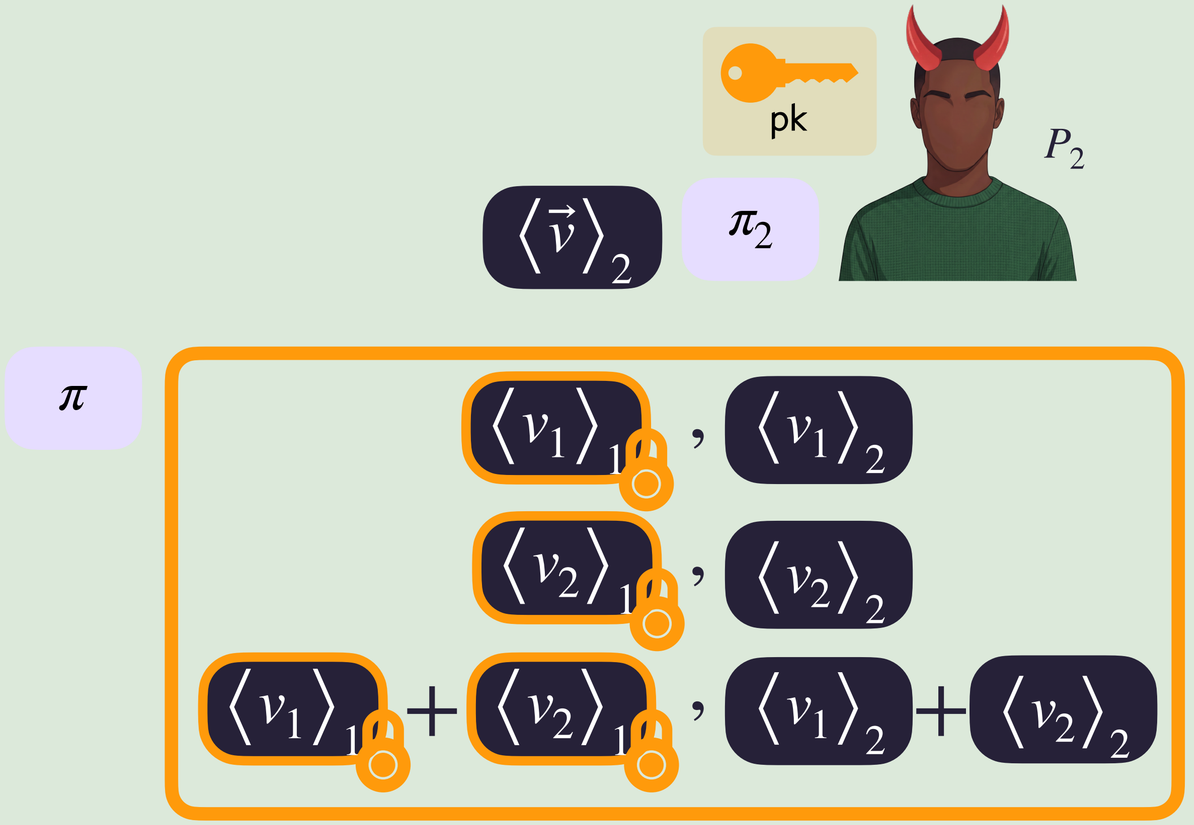
Selective Failure Attack

Problem:

P_2 can exploit linearity to test linear relations over $\vec{v}$

e.g.

test $v_1 + v_2 \stackrel{?}{=} v_3$



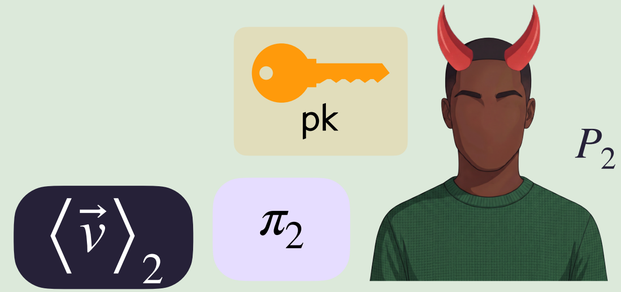
Selective Failure Attack

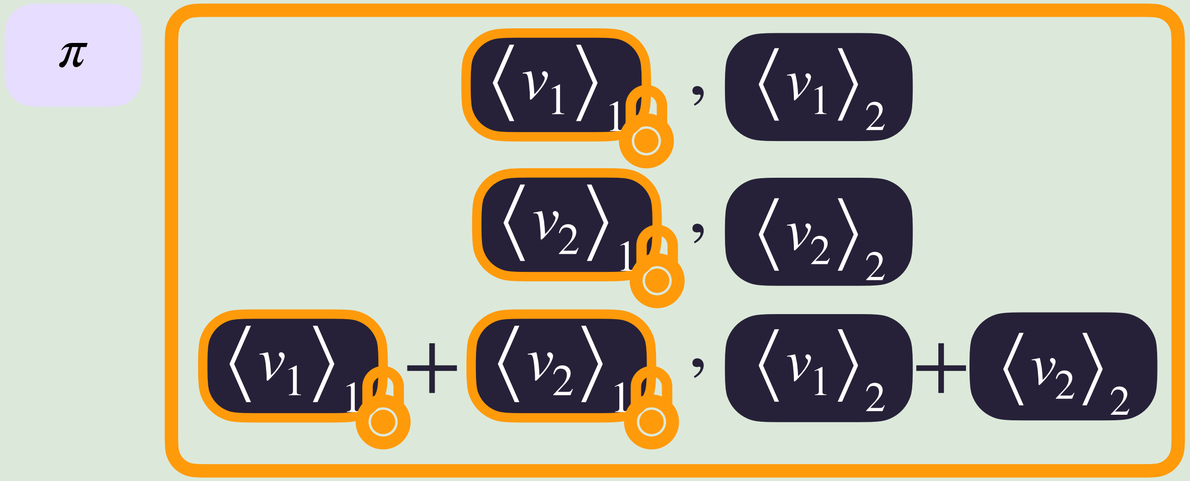
Problem:

P_2 can exploit linearity to test linear relations over $\vec{v}$

e.g.

test $v_1 + v_2 \stackrel{?}{=} v_3$

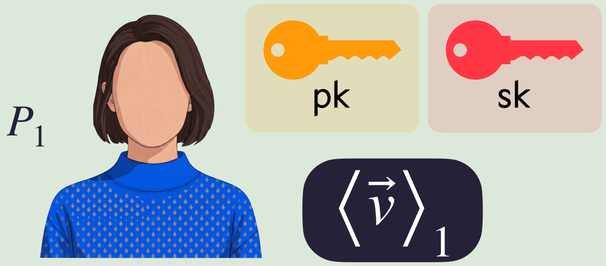


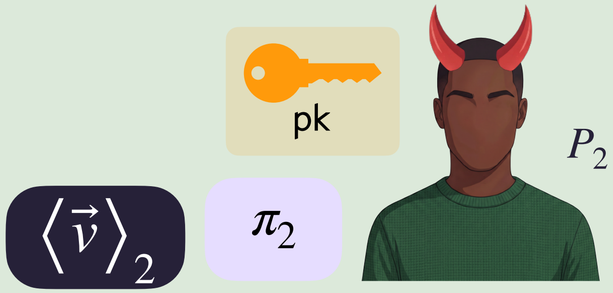


P_1 **aborts set equality test** $\implies v_1 + v_2 \neq v_3$

P_1 **does not abort** $\implies v_1 + v_2 = v_3$

Idea 2: Split Shares Randomly





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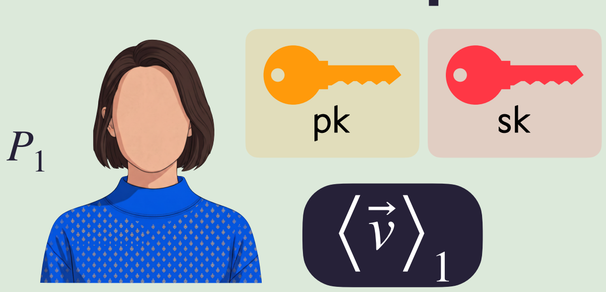
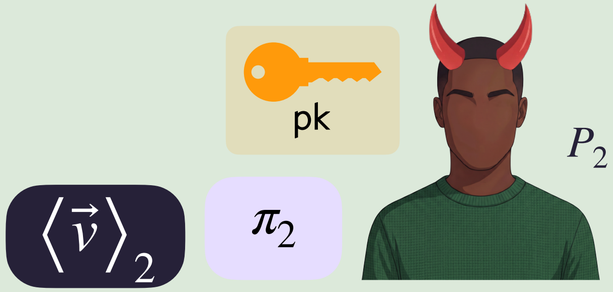


Sample shares of a **random masking vector** $\langle \vec{m} \rangle$

e.g. using SPDZ preprocessing



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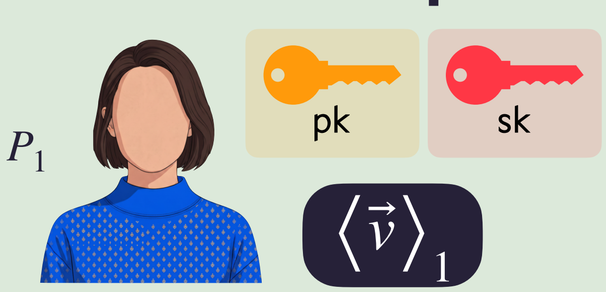

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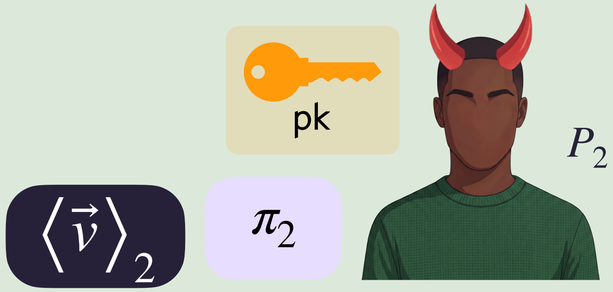
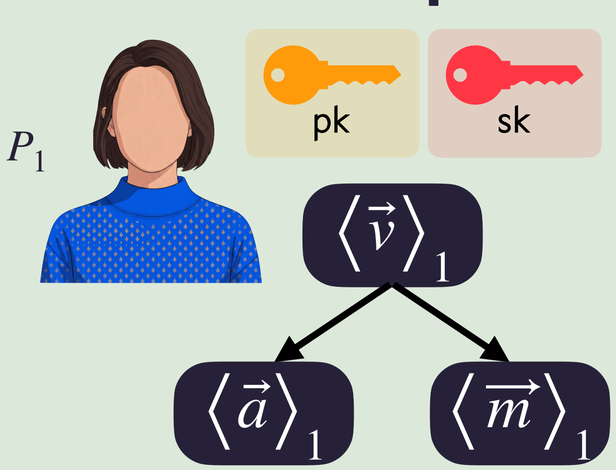


Compute $\langle \vec{a} \rangle \leftarrow \langle \vec{v} \rangle + \langle \vec{m} \rangle$

Idea 2: Split Shares Randomly



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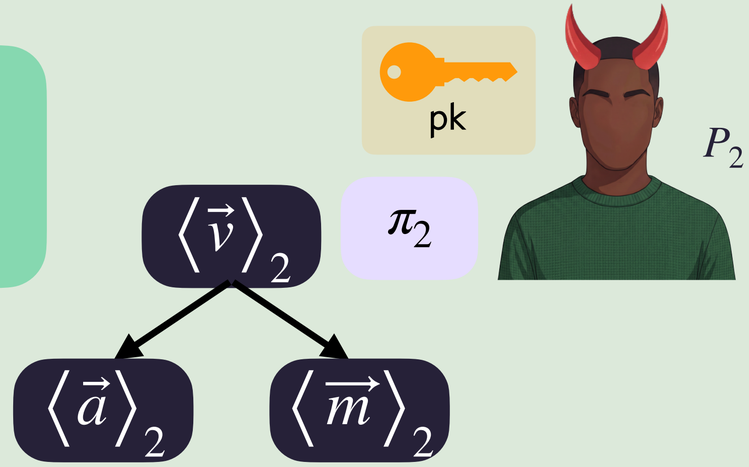

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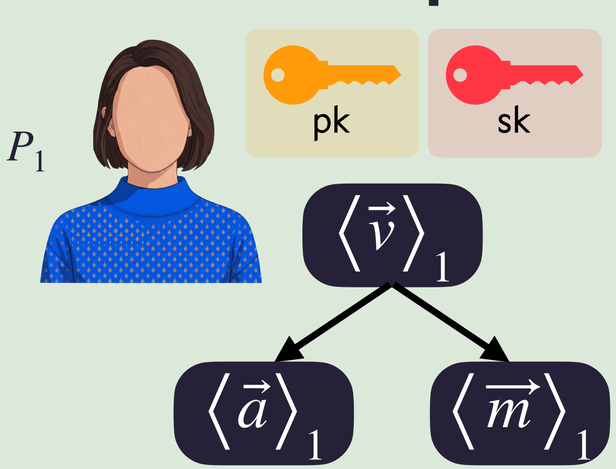

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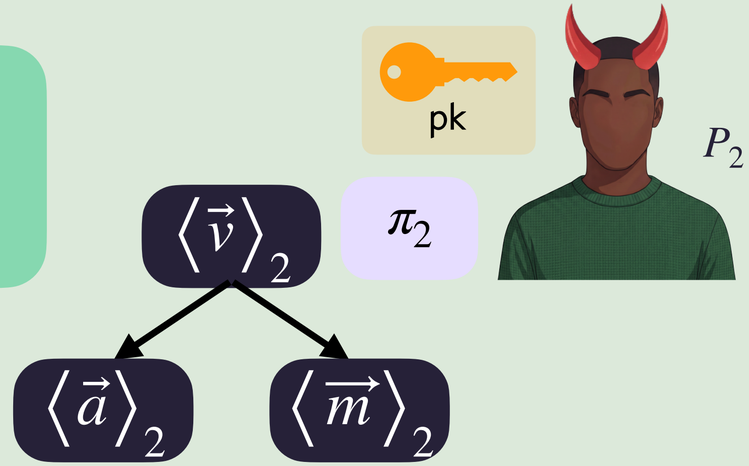
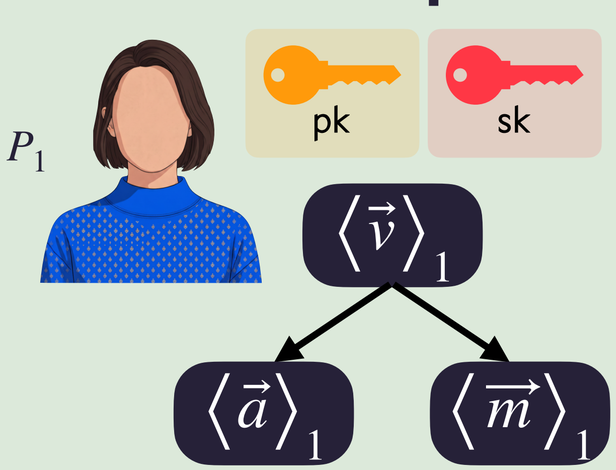


Do the protocol on $\vec{a}$ and $\vec{m}$ simultaneously instead of directly on $\vec{v}$;

in the end, reconstruct $\vec{w} = \pi_2(\vec{v})$

Idea 2: Split Shares Randomly



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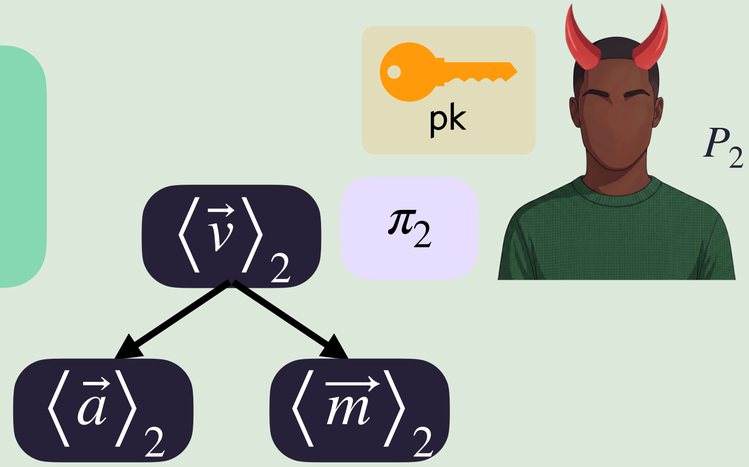

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Do the protocol on $\vec{a}$ and $\vec{m}$ simultaneously instead of directly on $\vec{v}$;

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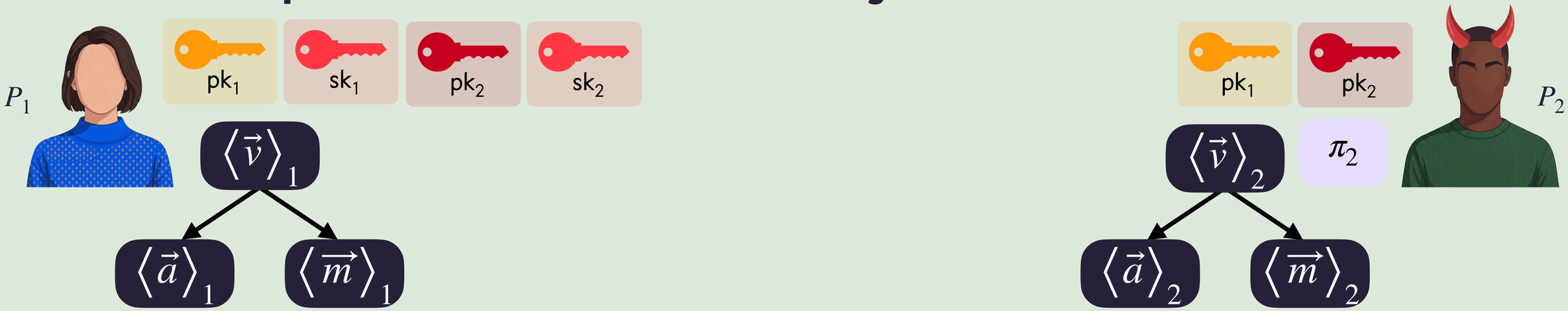
To prevent linear attacks, encrypt using ***different*** **public keys**

Idea 2: Split Shares Randomly



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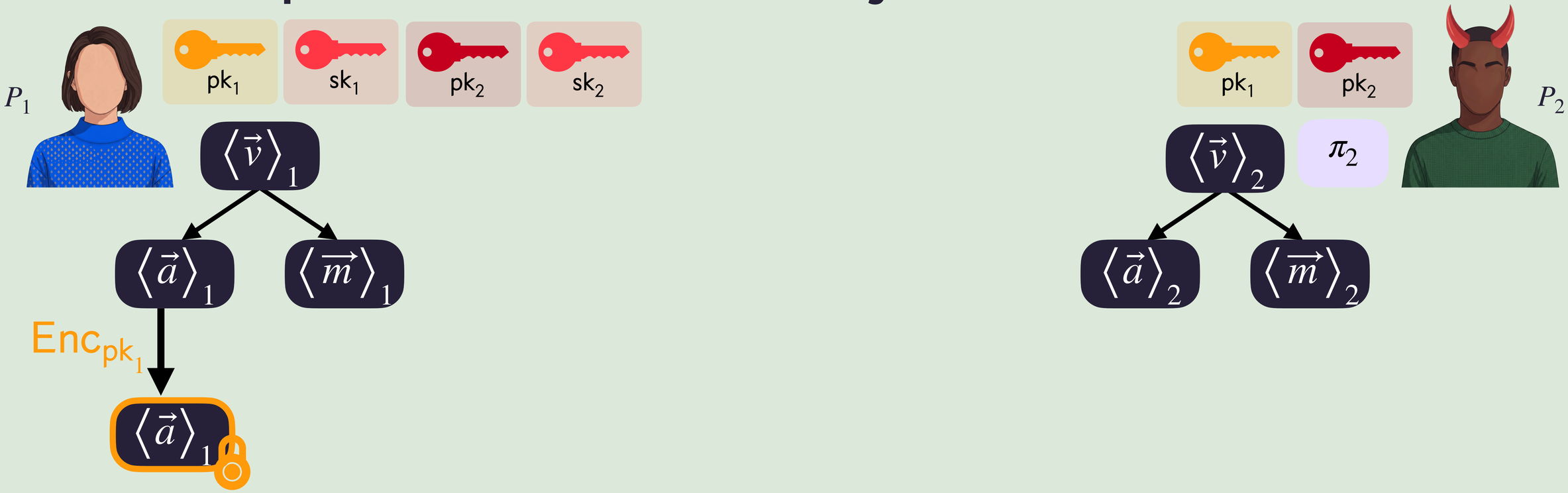

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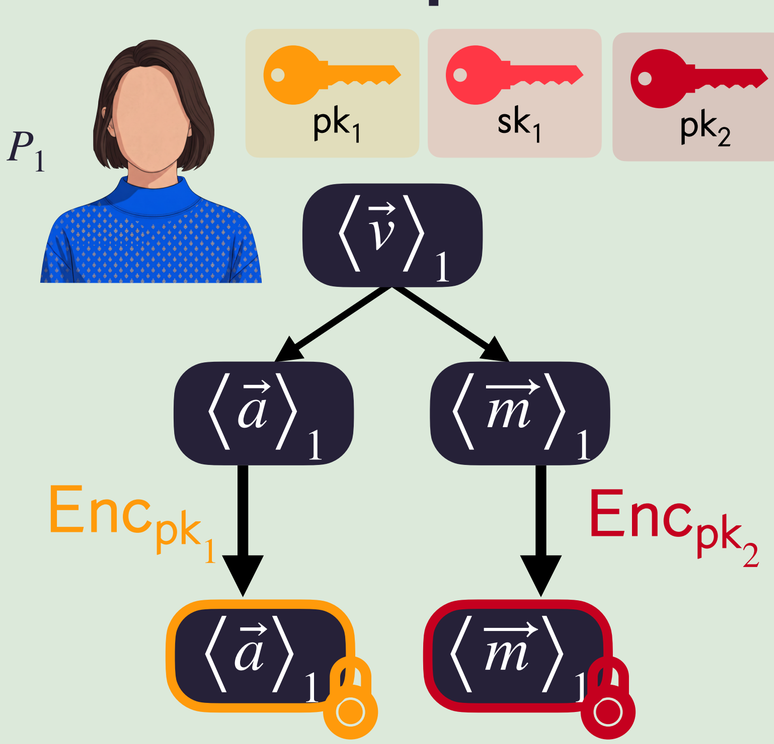
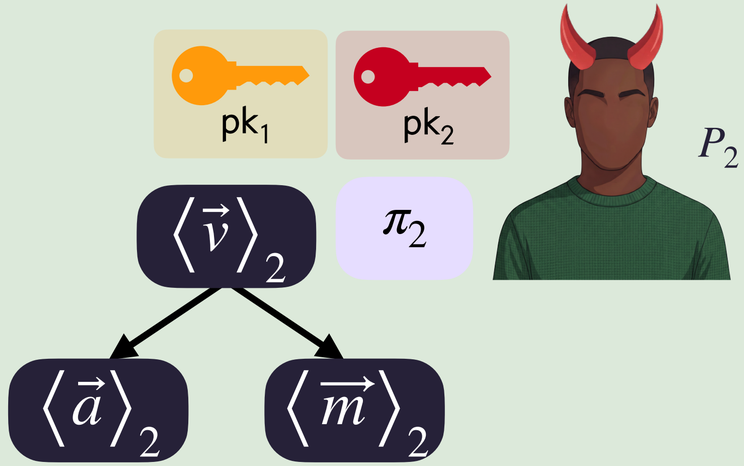
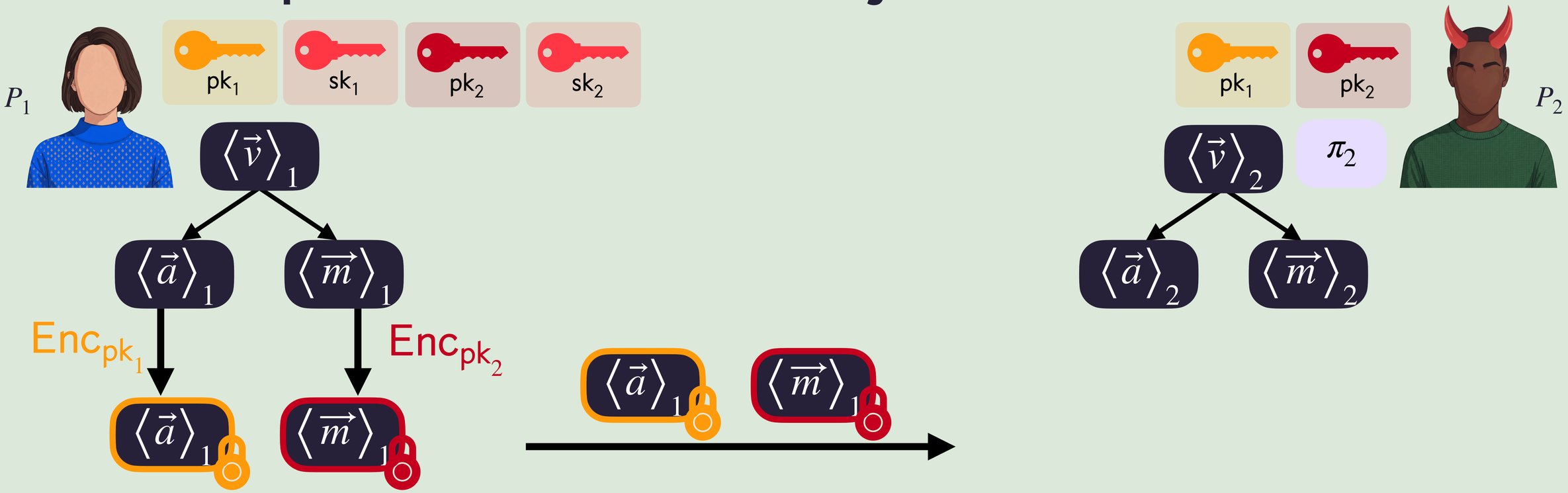




Idea 2: Split Shares Randomly



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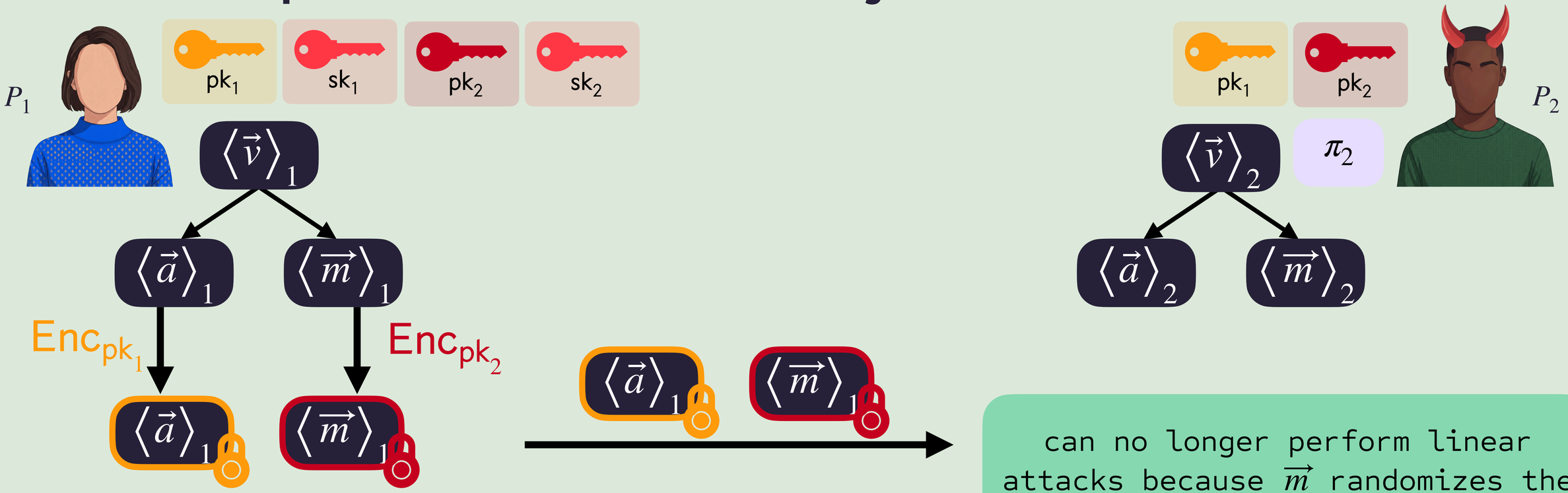
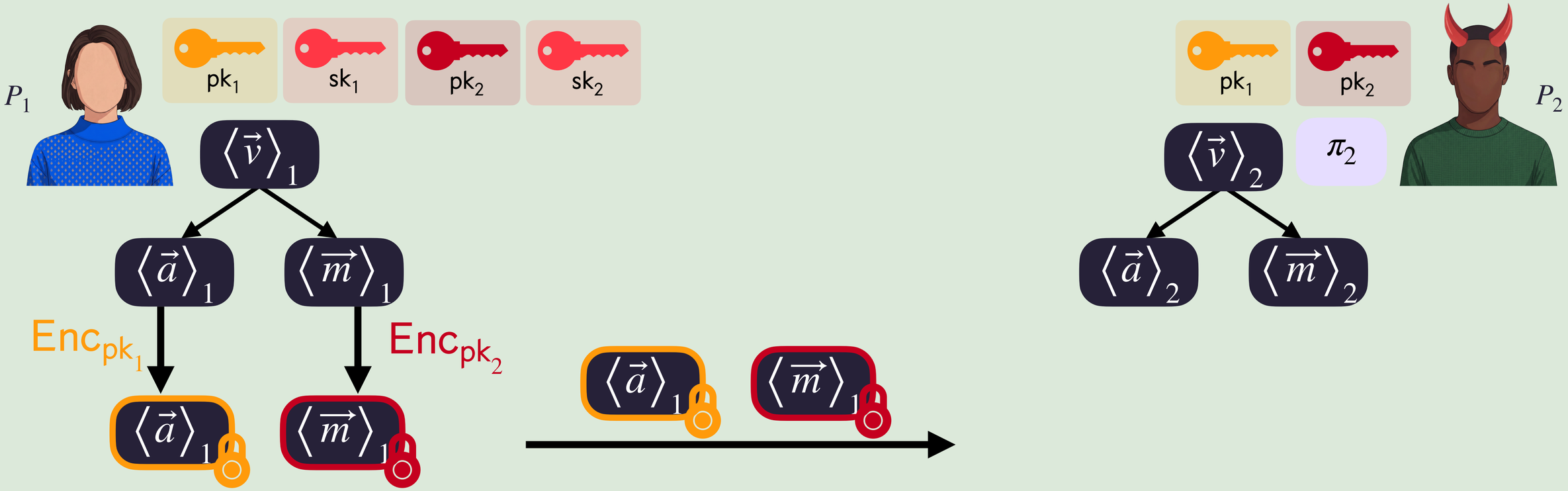


can no longer perform linear attacks because $\vec{m}$ randomizes the input $\vec{v}$, and ciphertexts of $\vec{a}$ and $\vec{m}$ are under different keys

What about non-linear attacks?



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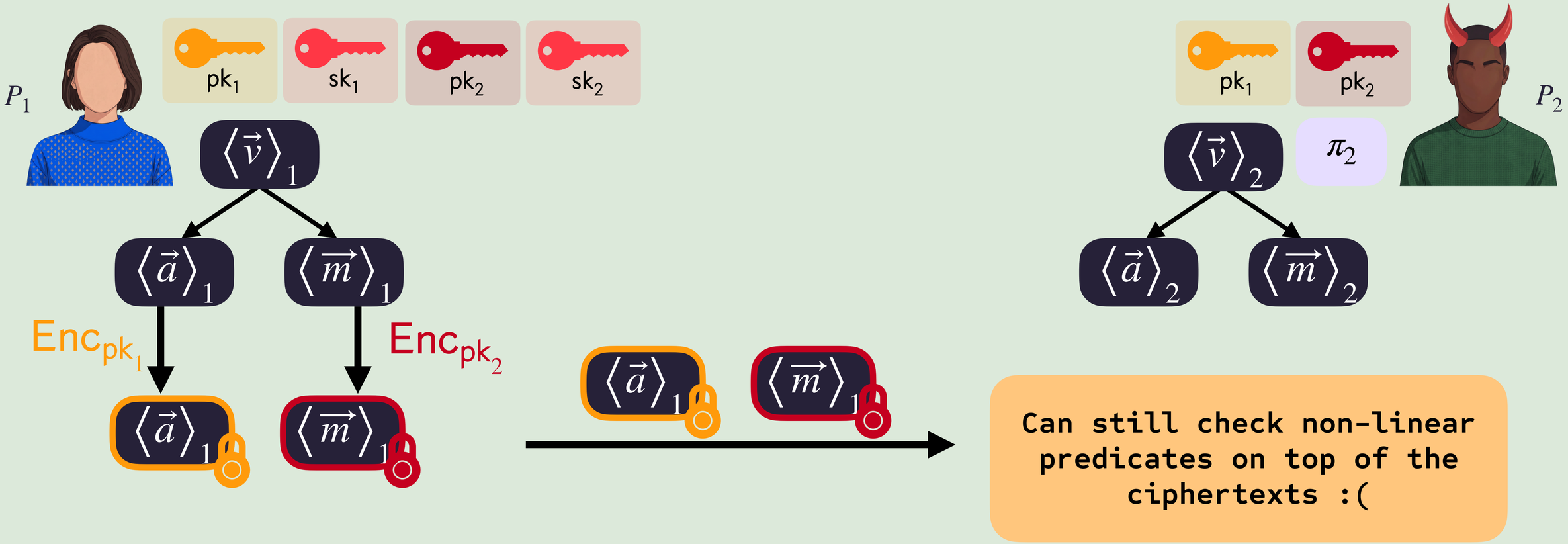

Can still check non-linear predicates on top of the ciphertexts :(

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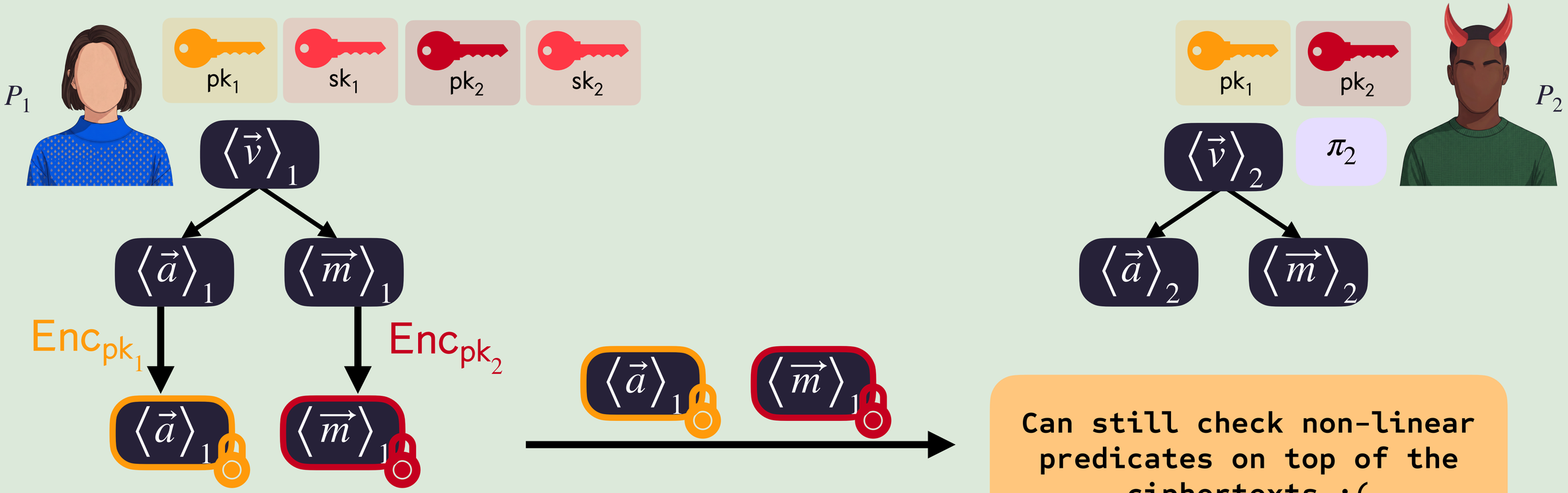

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To prevent non-linear predicates, we assume our LHE scheme is “**linear-only**”: a well-known (non-falsifiable) assumption in proof systems

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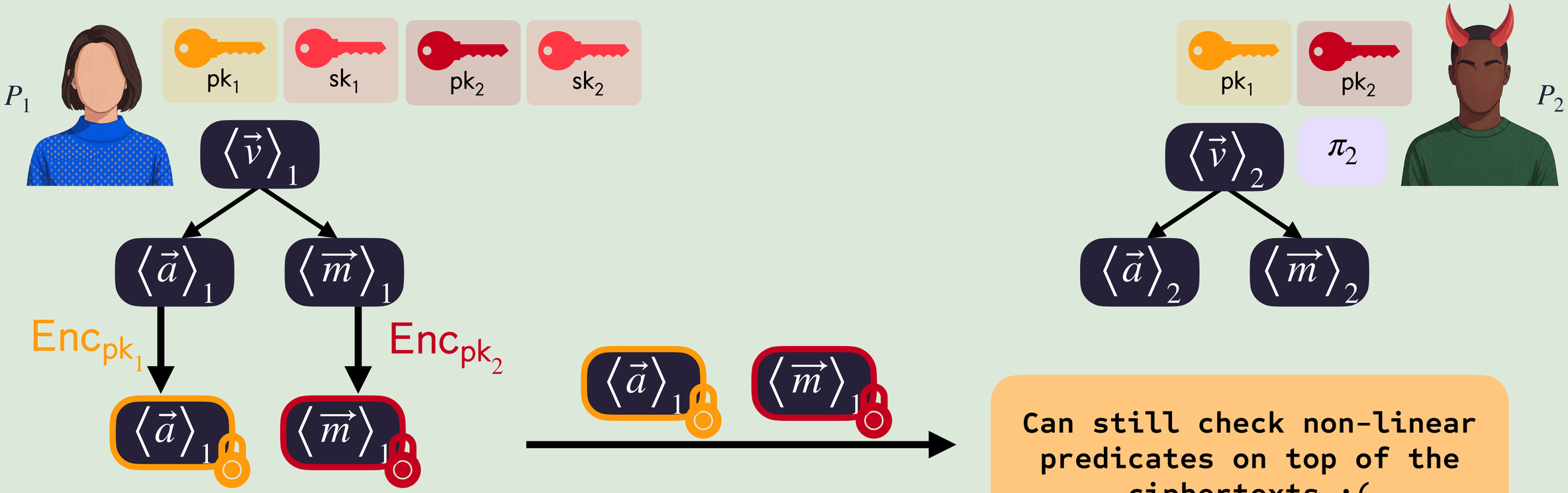

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linear targeted malleability

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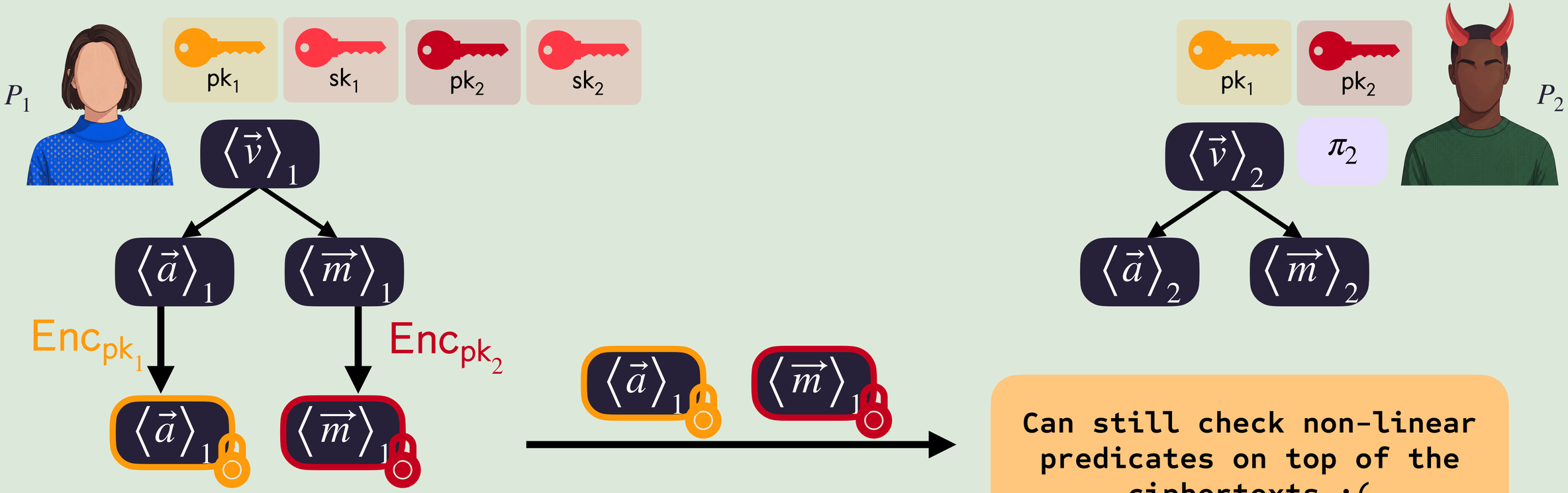

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- linear targeted malleability
- new: “weakly predicable”

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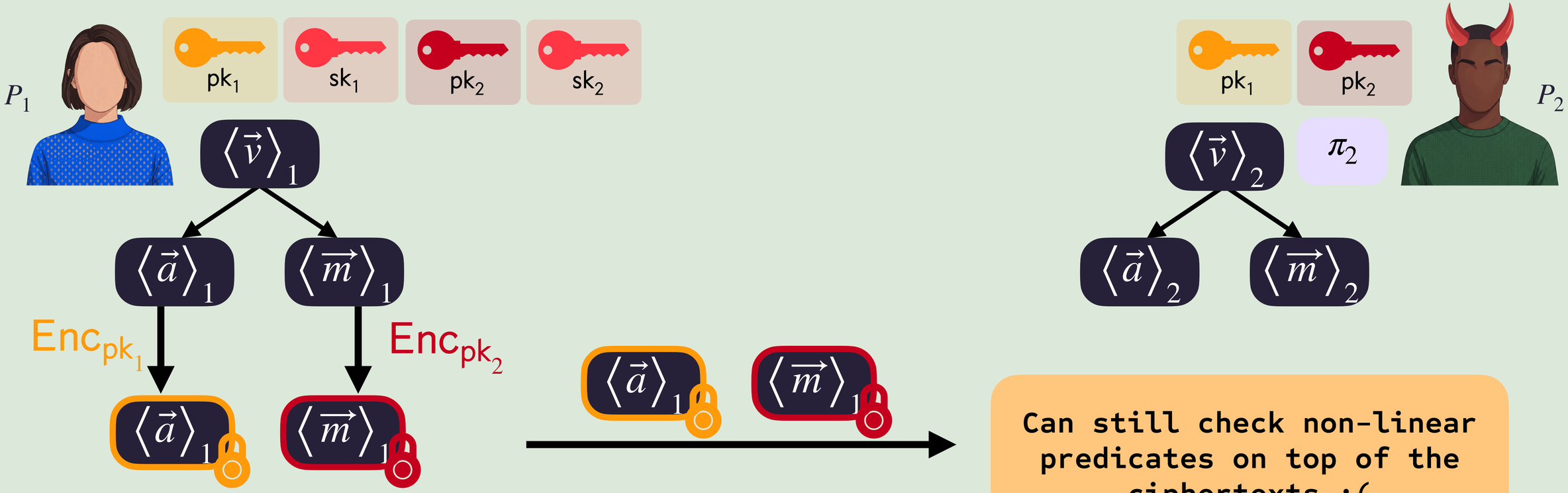

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weaker assumptions

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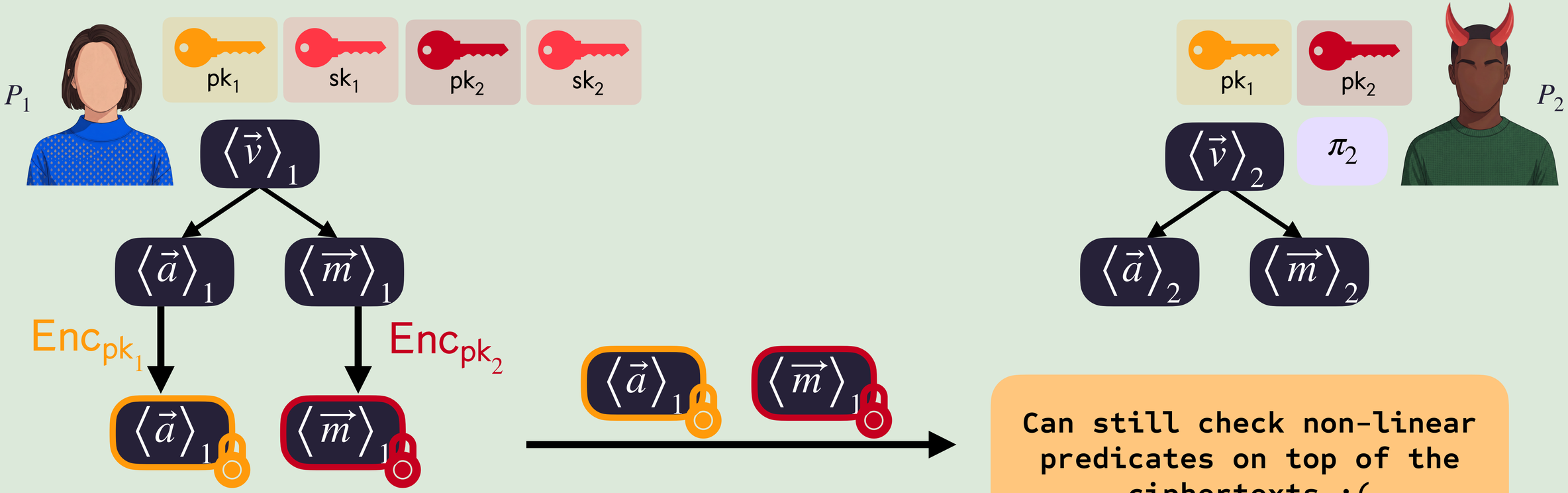

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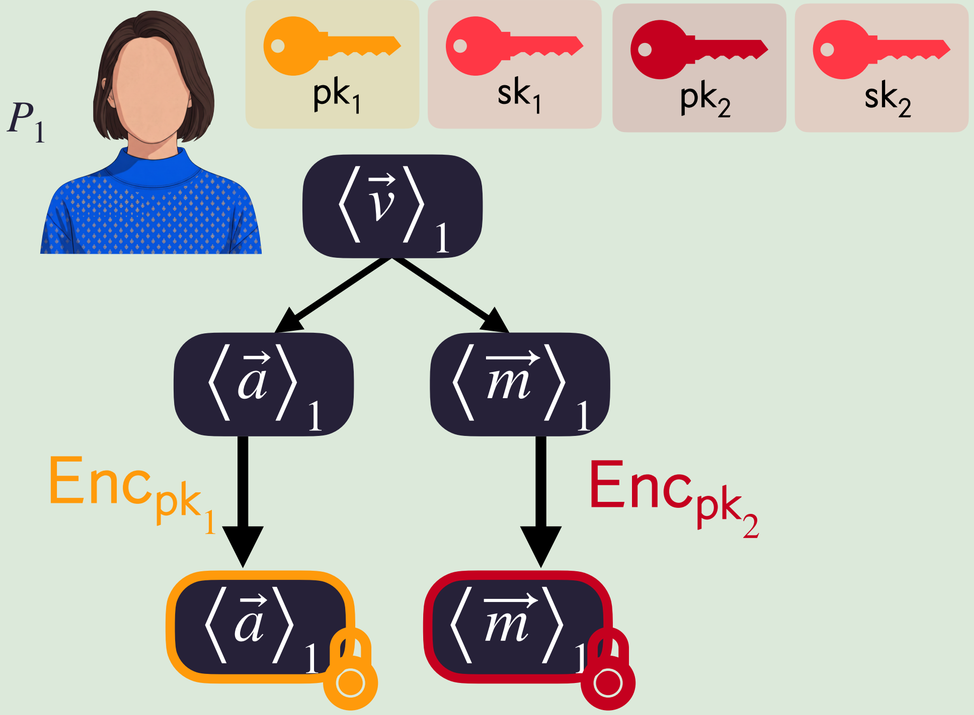
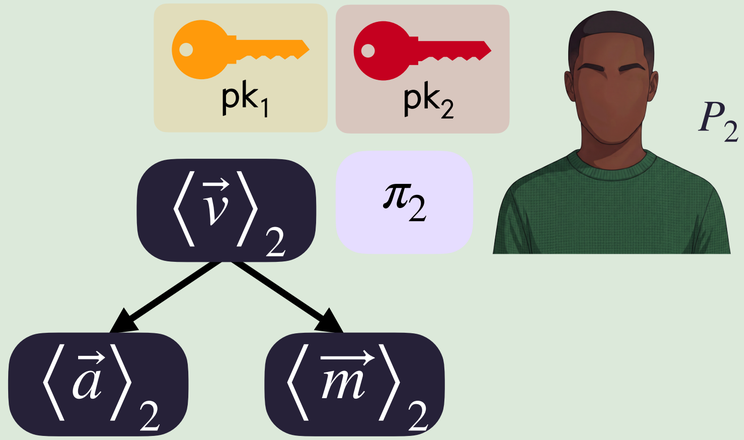
see paper for details!

weaker assumptions

Protocol so far...





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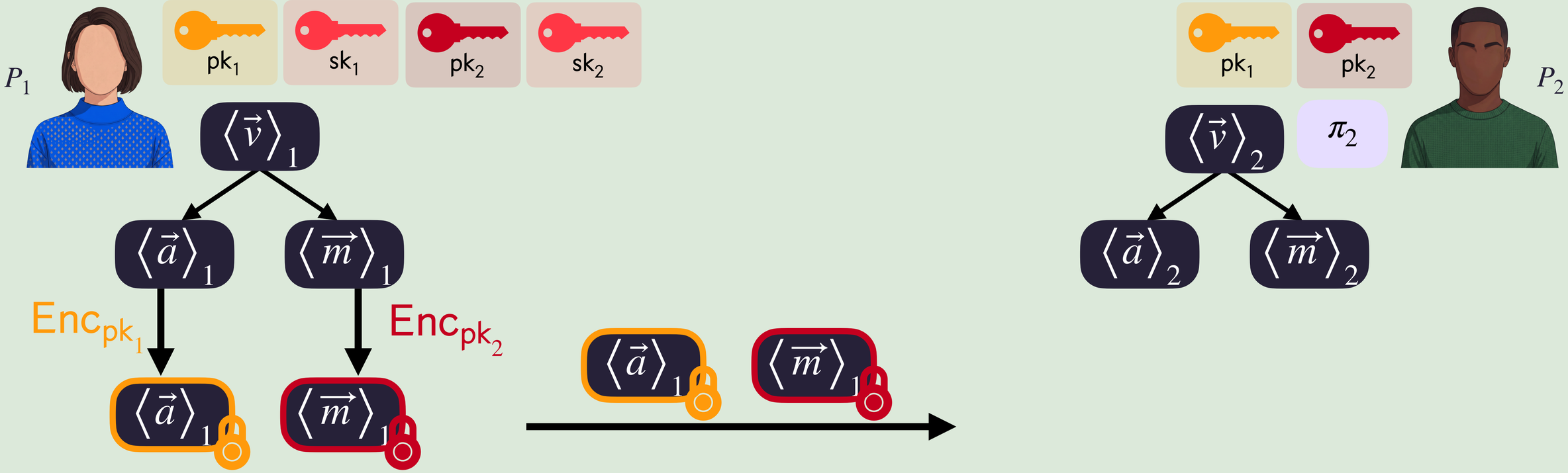

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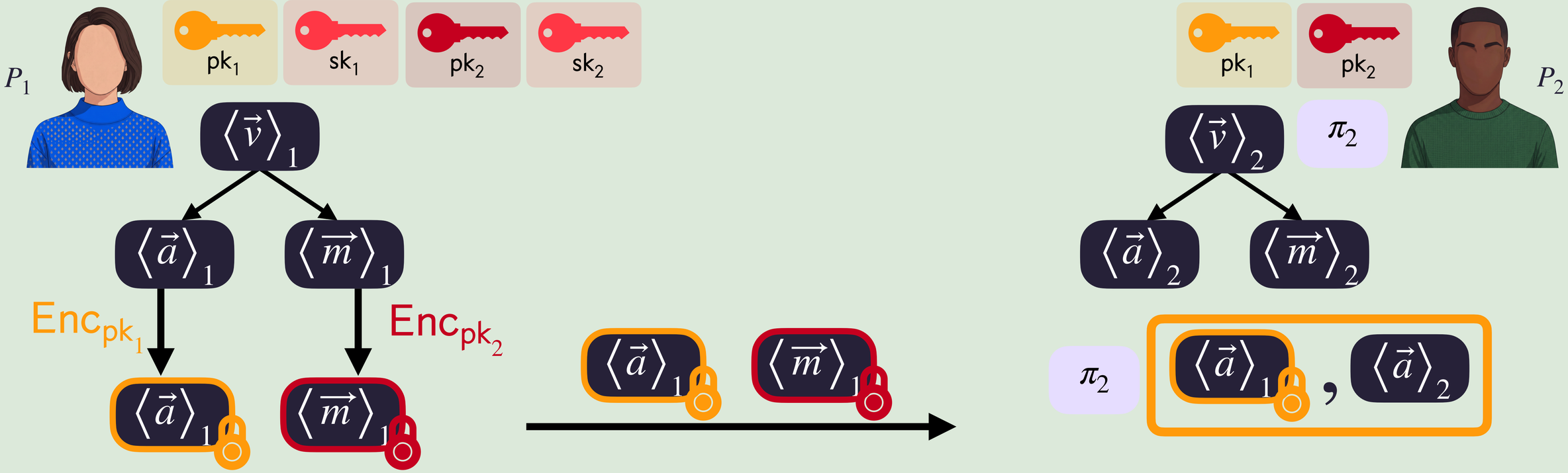

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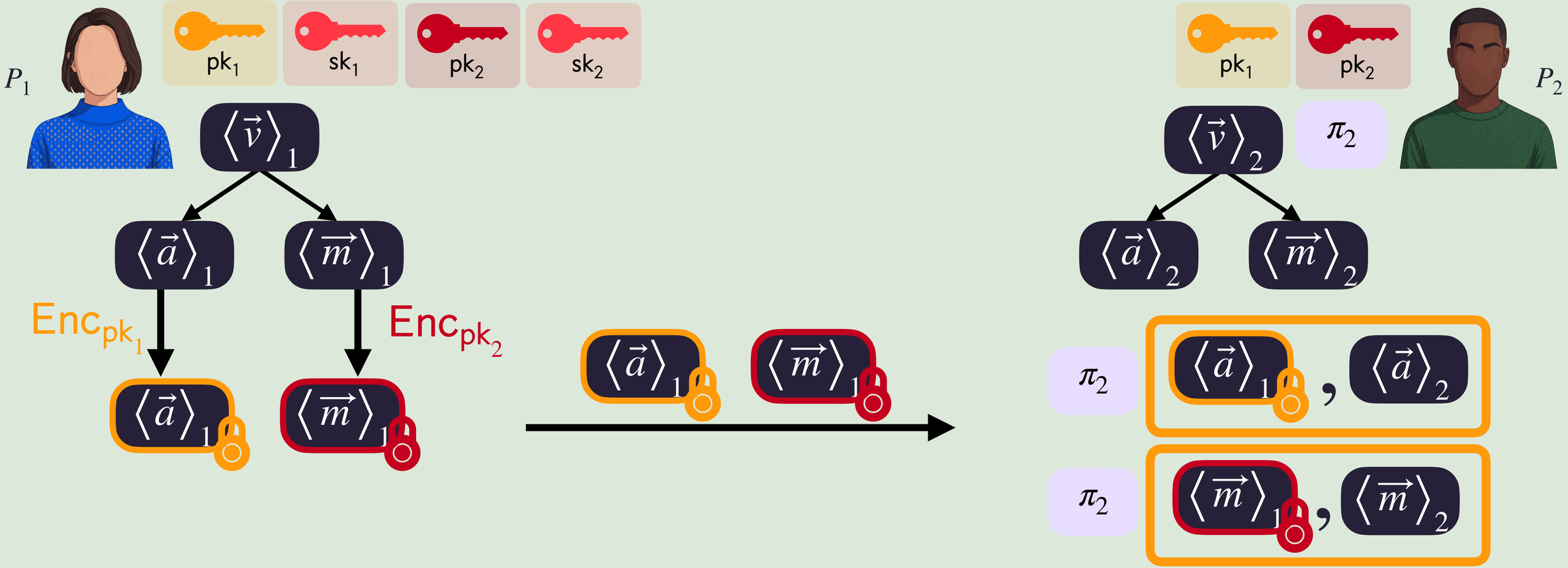

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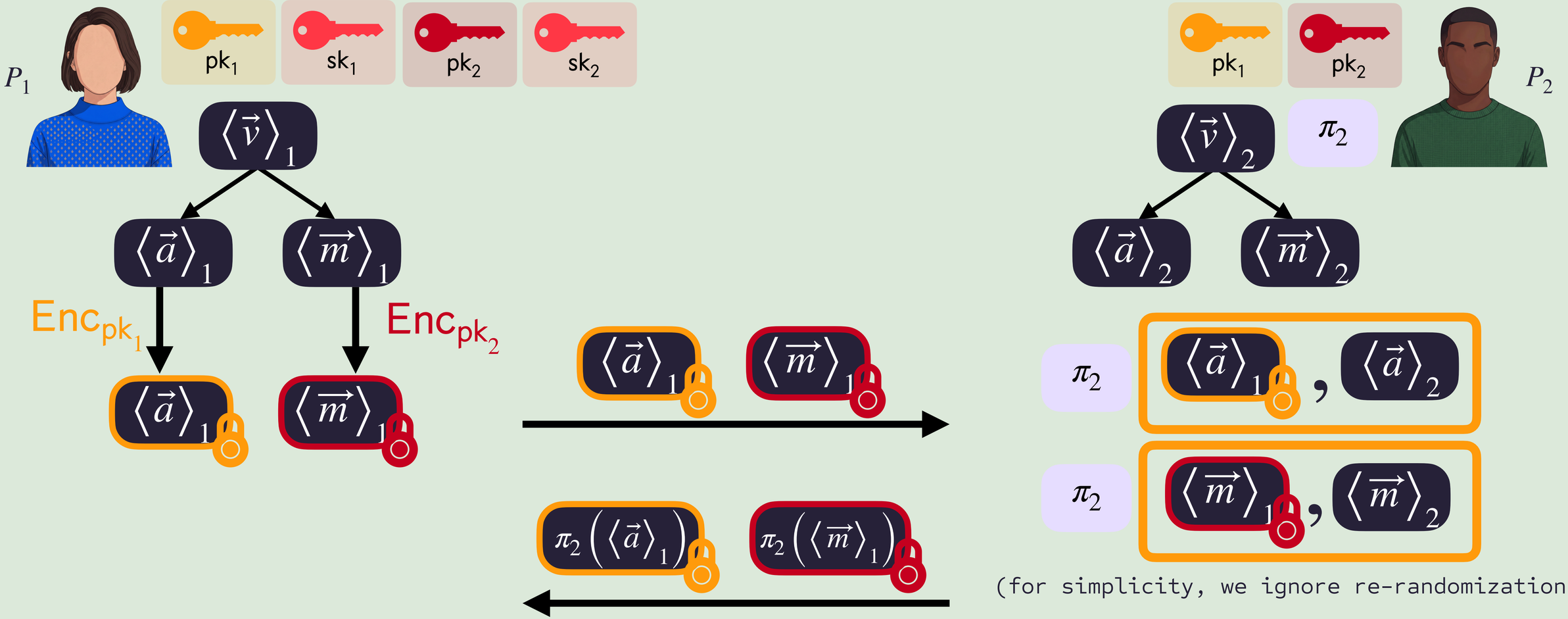

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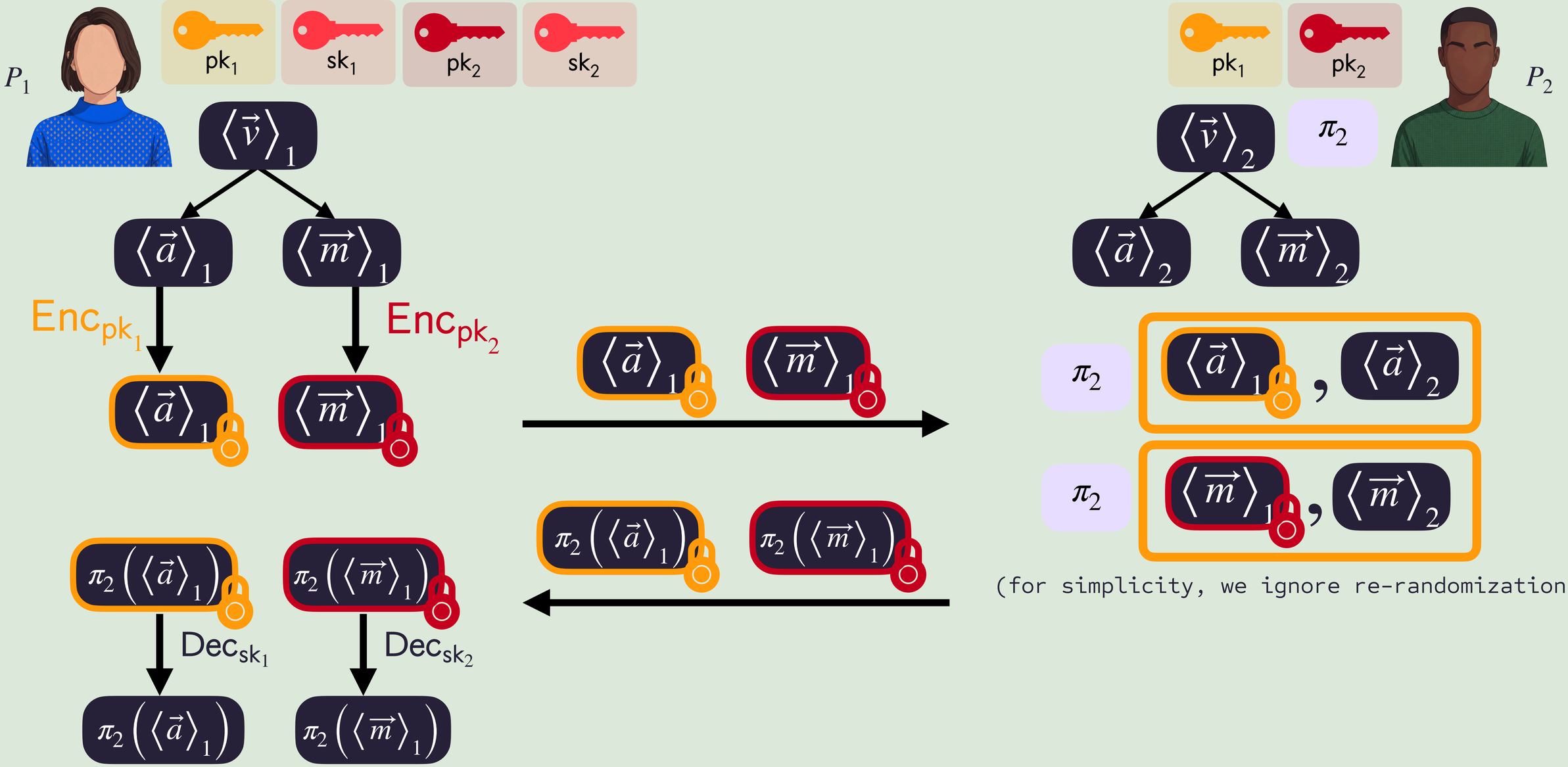
Protocol so far...



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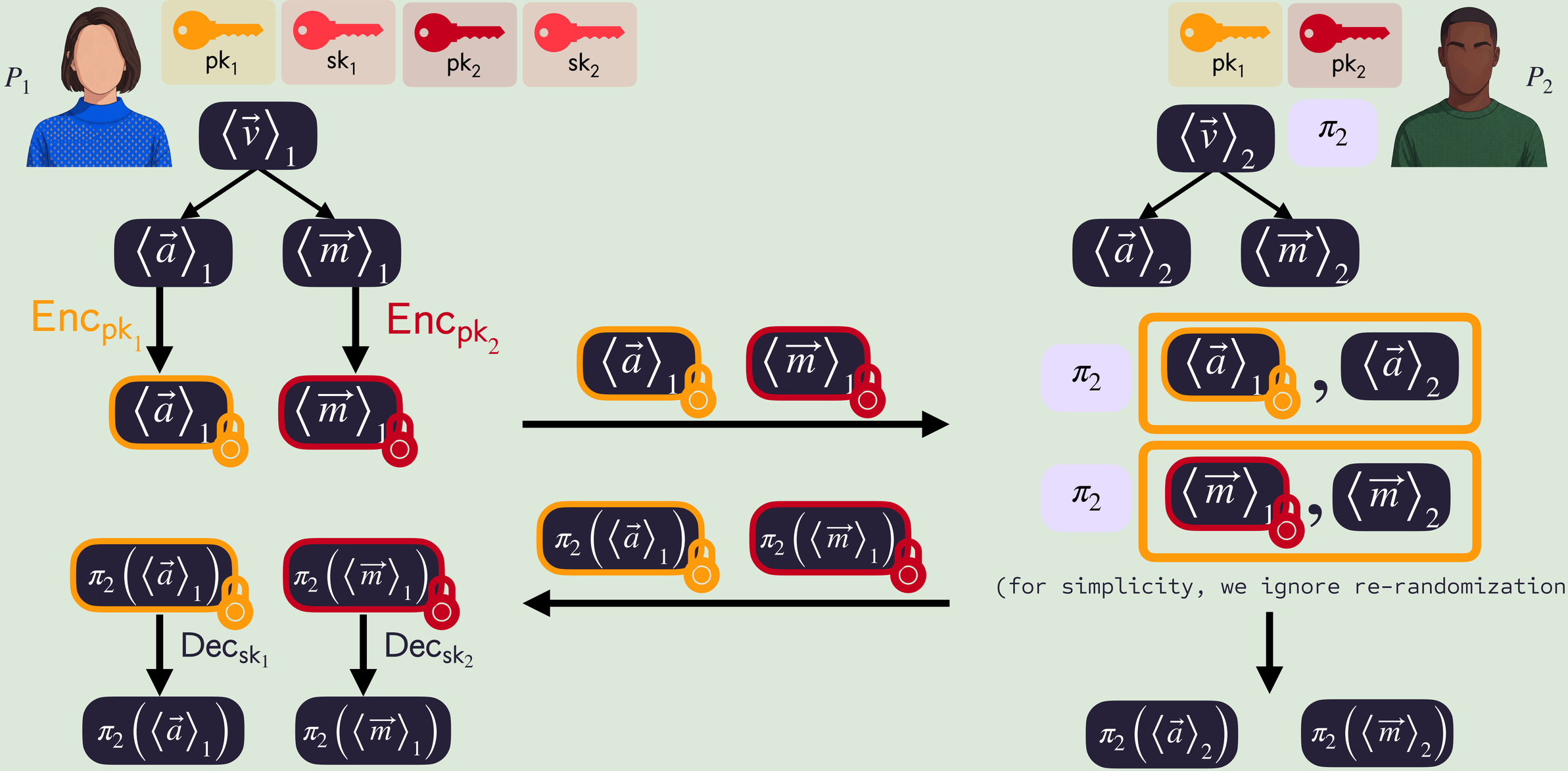

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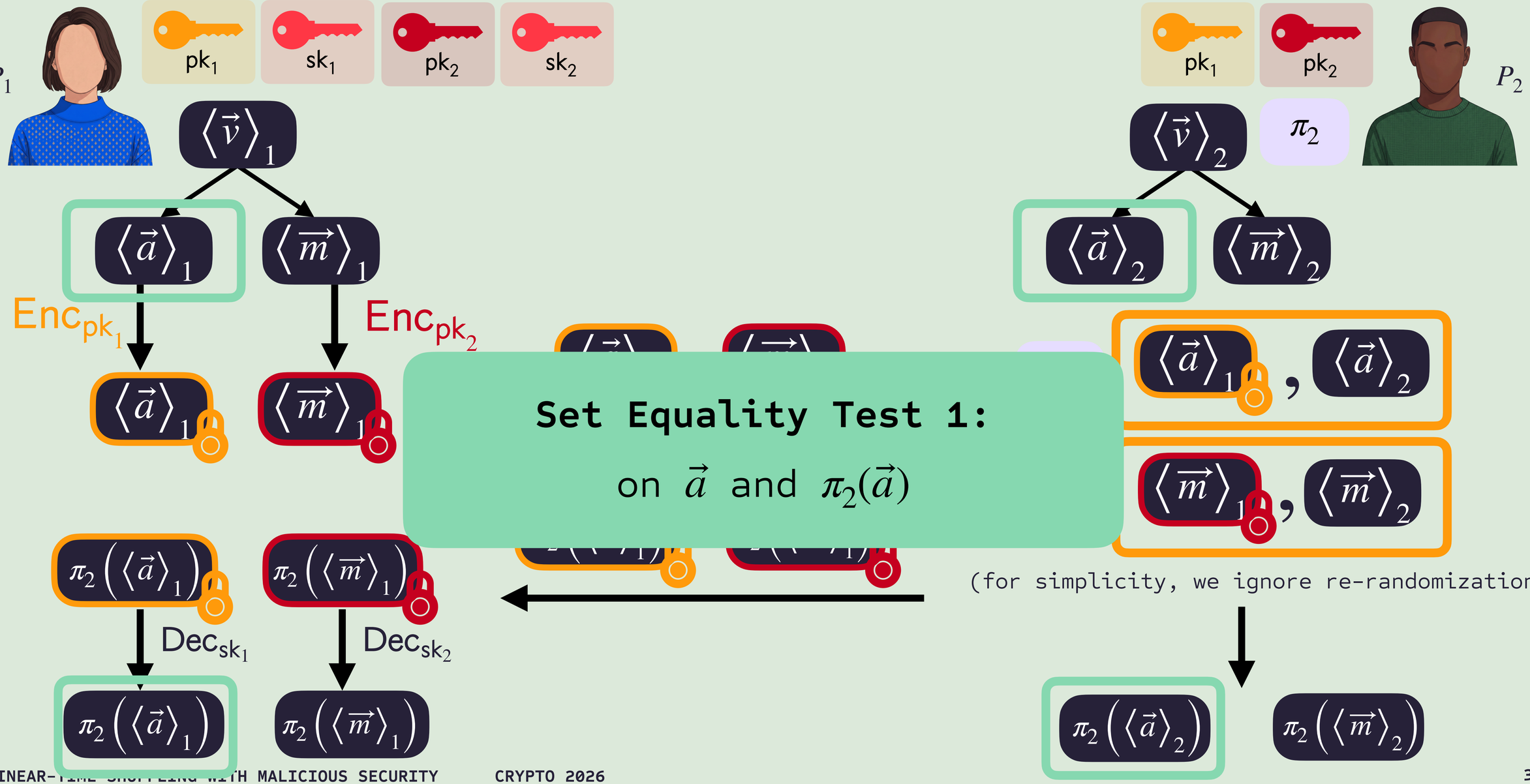
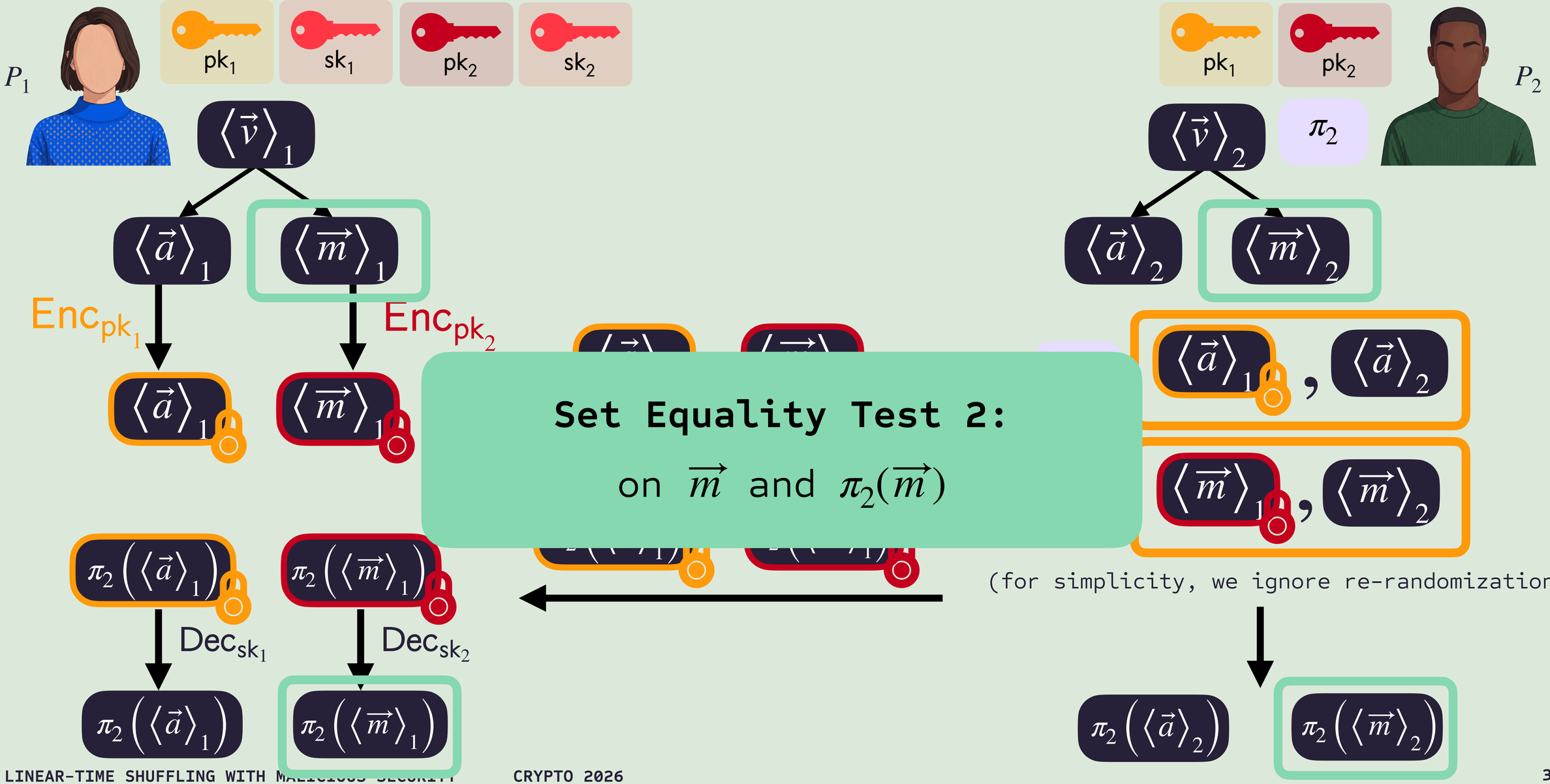


Set Equality Test 1:

on $\vec{a}$ and $\pi_2(\vec{a})$

(for simplicity, we ignore re-randomization)

Protocol so far...

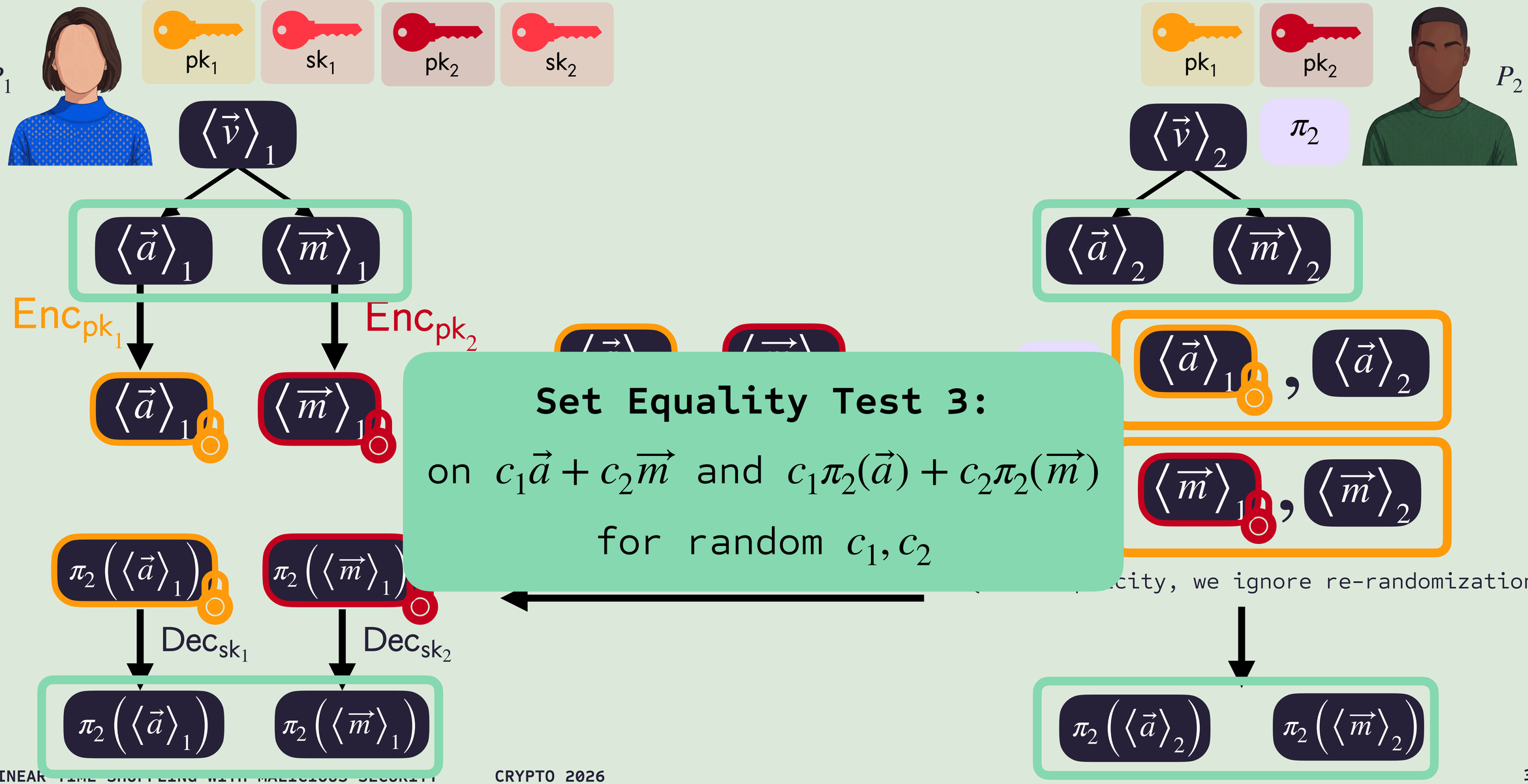


Set Equality Test 2:

on $\vec{m}$ and $\pi_2(\vec{m})$

(for simplicity, we ignore re-randomization)

Protocol so far...



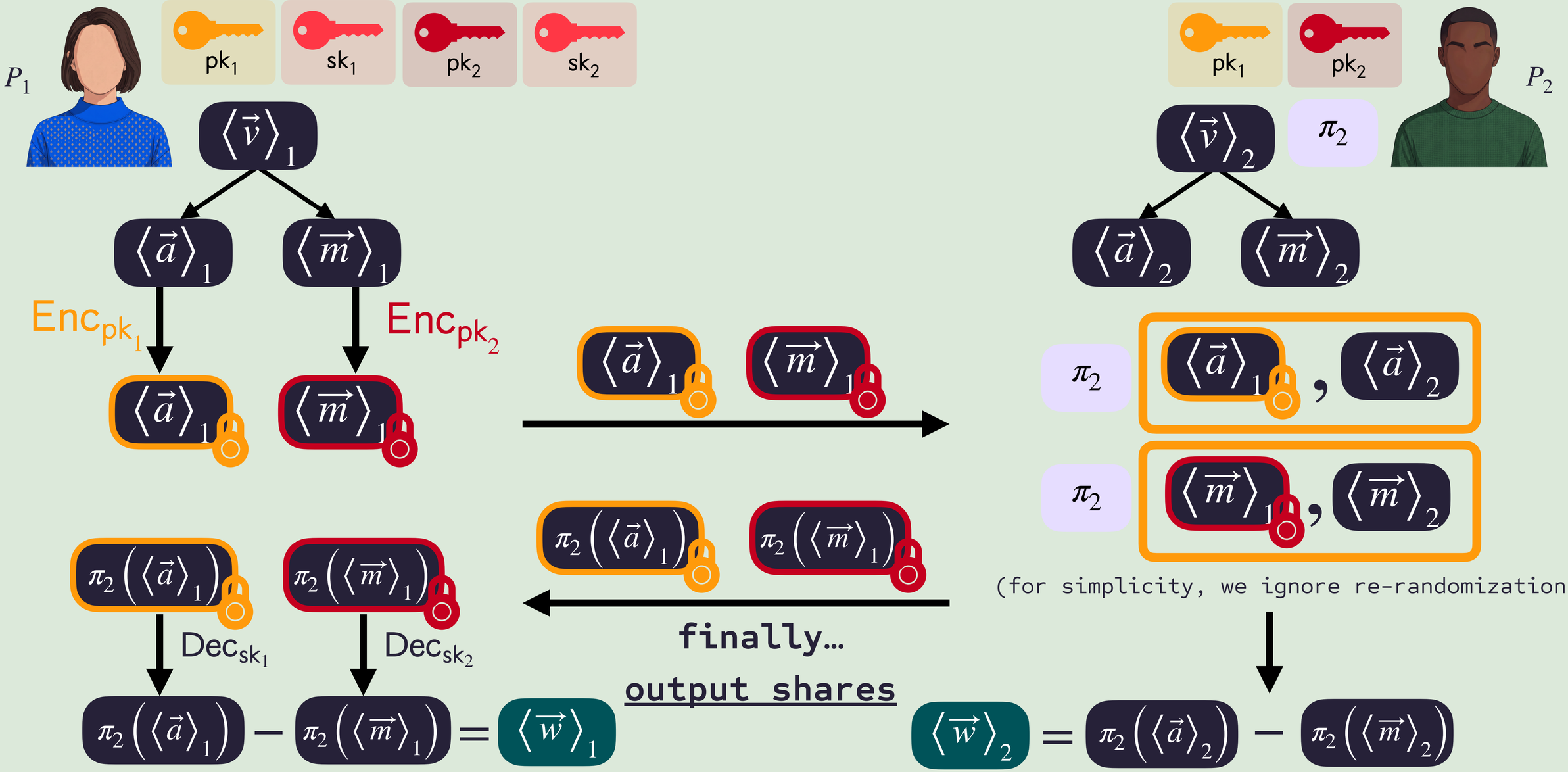
Set Equality Test 3:

on $c_1\vec{a} + c_2\vec{m}$ and $c_1\pi_2(\vec{a}) + c_2\pi_2(\vec{m})$

for random c_1, c_2

city, we ignore re-randomization)

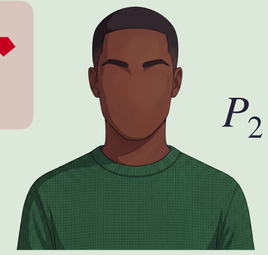
Protocol so far...



What about Malicious Encryptor P_1?

P_1

pk_1 sk_1 pk_2 sk_2



P_2

pk_1 pk_2

$\langle \vec{v} \rangle_1$ → $\langle \vec{a} \rangle_1$, $\langle \vec{m} \rangle_1$

$\mathsf{Enc}_{\mathsf{pk}_1}$ $\langle \vec{a} \rangle_1$

$\mathsf{Enc}_{\mathsf{pk}_2}$ $\langle \vec{m} \rangle_1$

$\langle \vec{a} \rangle_1$ $\langle \vec{m} \rangle_1$ →

$\langle \vec{v} \rangle_2$ → $\langle \vec{a} \rangle_2$, $\langle \vec{m} \rangle_2$

π_2

π_2 $\langle \vec{a} \rangle_1$, $\langle \vec{a} \rangle_2$

π_2 $\langle \vec{m} \rangle_1$, $\langle \vec{m} \rangle_2$

(for simplicity, we ignore re-randomization)

$\pi_2(\langle \vec{a} \rangle_1)$ $\pi_2(\langle \vec{m} \rangle_1)$ ←

finally…

output shares

$\mathsf{Dec}_{\mathsf{sk}_1}$ $\pi_2(\langle \vec{a} \rangle_1)$

$\mathsf{Dec}_{\mathsf{sk}_2}$ $\pi_2(\langle \vec{m} \rangle_1)$

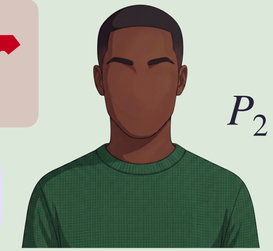
$\pi_2(\langle \vec{a} \rangle_1) - \pi_2(\langle \vec{m} \rangle_1) = \langle \vec{w} \rangle_1$

$\langle \vec{w} \rangle_2 = \pi_2(\langle \vec{a} \rangle_2) - \pi_2(\langle \vec{m} \rangle_2)$

What about Malicious Encryptor P_1?

P_1

pk_1 sk_1 pk_2 sk_2



P_2

pk_1 pk_2

$\langle \vec{v} \rangle_1$

$\langle \vec{v} \rangle_2$ π_2

$\langle \vec{a} \rangle_1$ $\langle \vec{m} \rangle_1$

$\mathsf{Enc}_{\mathsf{pk}_1}$ $\mathsf{Enc}_{\mathsf{pk}_2}$

$\langle \vec{a} \rangle_1$ $\langle \vec{m} \rangle_1$

$\langle \vec{a} \rangle_2$ $\langle \vec{m} \rangle_2$

P_1 can encrypt to incorrect shares

$\langle \vec{a} \rangle_1$ $\langle \vec{m} \rangle_1$

π_2 $\langle \vec{a} \rangle_1$, $\langle \vec{a} \rangle_2$

π_2 $\langle \vec{m} \rangle_1$, $\langle \vec{m} \rangle_2$

$\pi_2(\langle \vec{a} \rangle_1)$ $\pi_2(\langle \vec{m} \rangle_1)$

(for simplicity, we ignore re-randomization)

$\pi_2(\langle \vec{a} \rangle_1)$ $\pi_2(\langle \vec{m} \rangle_1)$

$\mathsf{Dec}_{\mathsf{sk}_1}$ $\mathsf{Dec}_{\mathsf{sk}_2}$

finally…

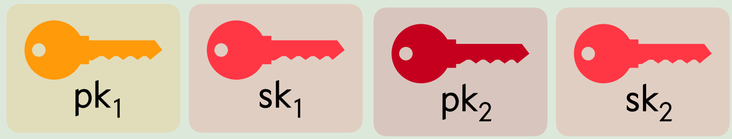
output shares

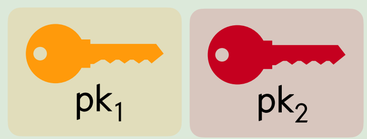
$\pi_2(\langle \vec{a} \rangle_1) - \pi_2(\langle \vec{m} \rangle_1) = \langle \vec{w} \rangle_1$

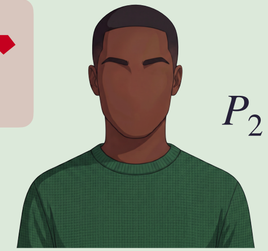
$\langle \vec{w} \rangle_2 = \pi_2(\langle \vec{a} \rangle_2) - \pi_2(\langle \vec{m} \rangle_2)$

What about Malicious Encryptor P_1?

P_1







P_2

$\langle \vec{v} \rangle_1$

$\langle \vec{a} \rangle_1$ $\langle \vec{m} \rangle_1$

$\mathsf{Enc}_{\mathsf{pk}_1}$ $\mathsf{Enc}_{\mathsf{pk}_2}$

$\langle \vec{a} \rangle_1$ $\langle \vec{m} \rangle_1$

$\langle \vec{v} \rangle_2$ π_2

$\langle \vec{a} \rangle_2$ $\langle \vec{m} \rangle_2$

P_1 can encrypt to incorrect shares

concrete attack: P_1 can learn information about the permutation with non-negligible probability

π_2 $\langle \vec{a} \rangle_1, \langle \vec{a} \rangle_2$

π_2 $\langle \vec{m} \rangle_1, \langle \vec{m} \rangle_2$

$\pi_2(\langle \vec{a} \rangle_1)$ $\pi_2(\langle \vec{m} \rangle_1)$

$\mathsf{Dec}_{\mathsf{sk}_1}$ $\mathsf{Dec}_{\mathsf{sk}_2}$

$\pi_2(\langle \vec{a} \rangle_1) - \pi_2(\langle \vec{m} \rangle_1) = \langle \vec{w} \rangle_1$

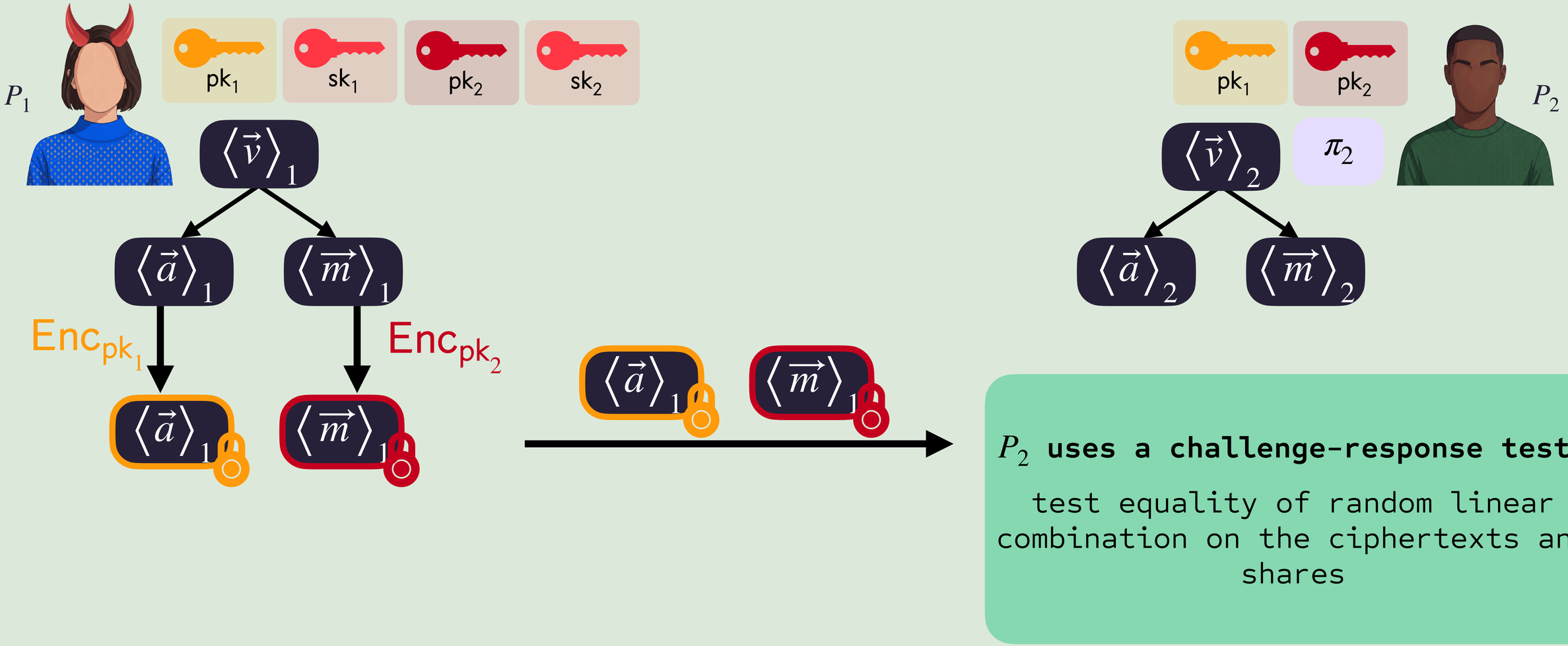
finally…

output shares

(for simplicity, we ignore re-randomization)

$\langle \vec{w} \rangle_2 = \pi_2(\langle \vec{a} \rangle_2) - \pi_2(\langle \vec{m} \rangle_2)$

Idea 3: Correlation Checks



P_2 uses a challenge-response test:

test equality of random linear combination on the ciphertexts and shares

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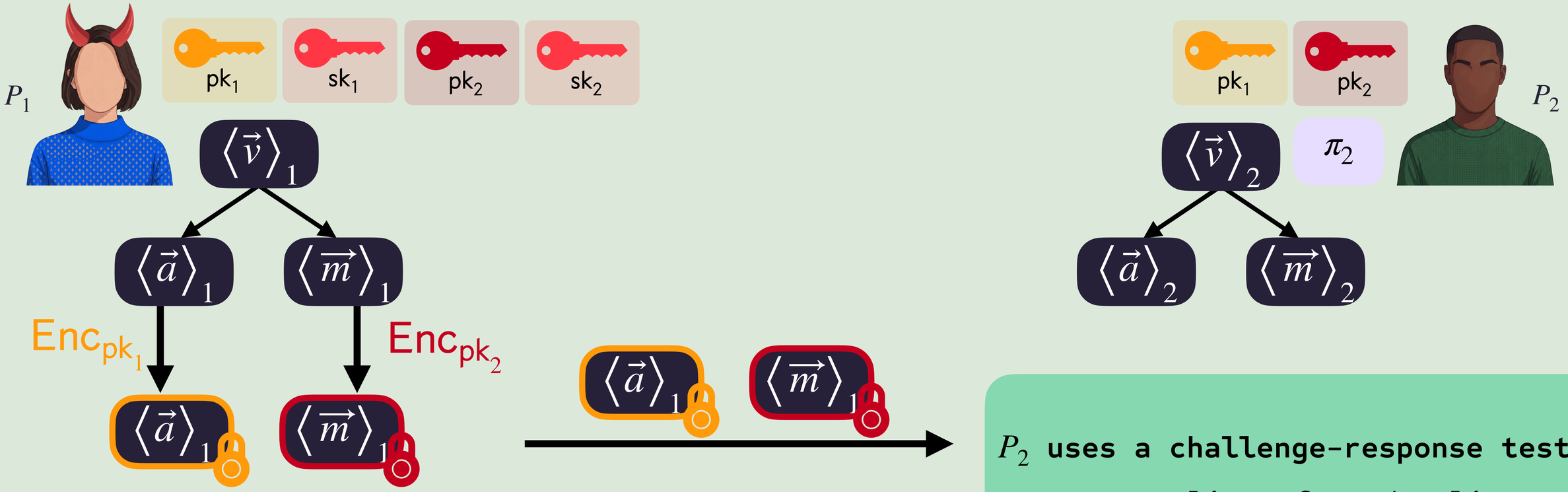

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see paper for details

Recap

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Folklore protocol using Linearly Homomorphic Encryption (LHE) and authenticated secret sharing

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(2) Prevent tampering of shares

(a) using authenticated secret sharing,

(b) splitting shares under different keys, and

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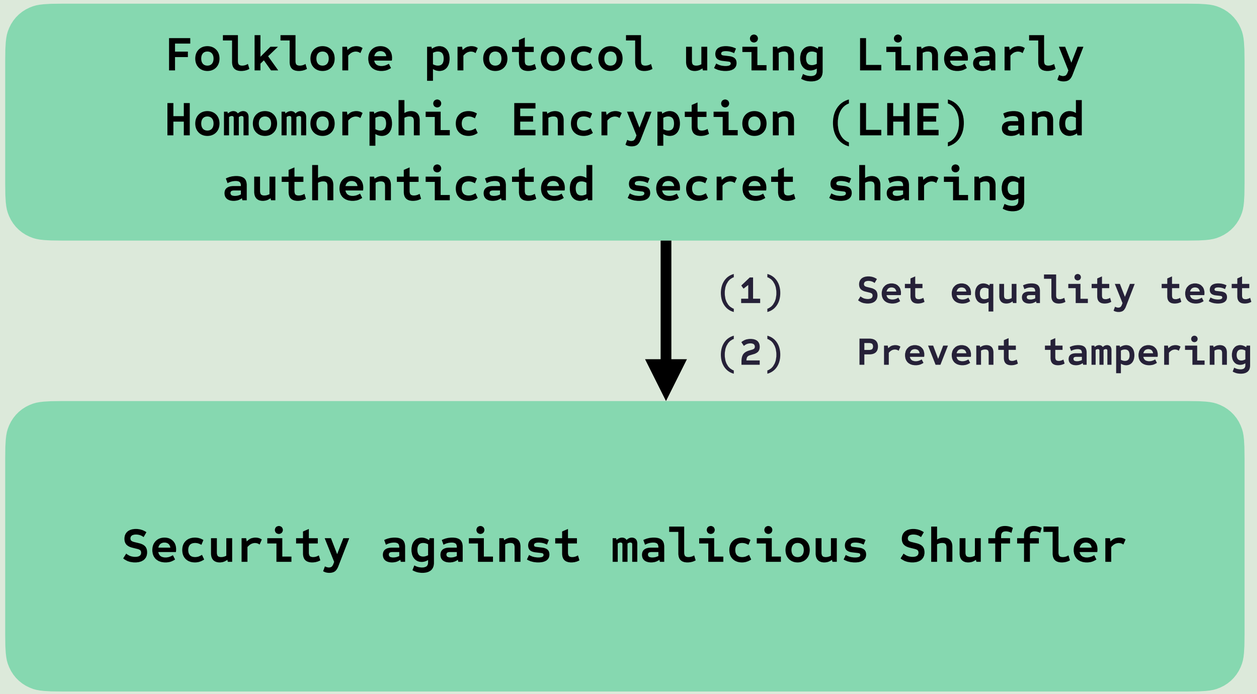
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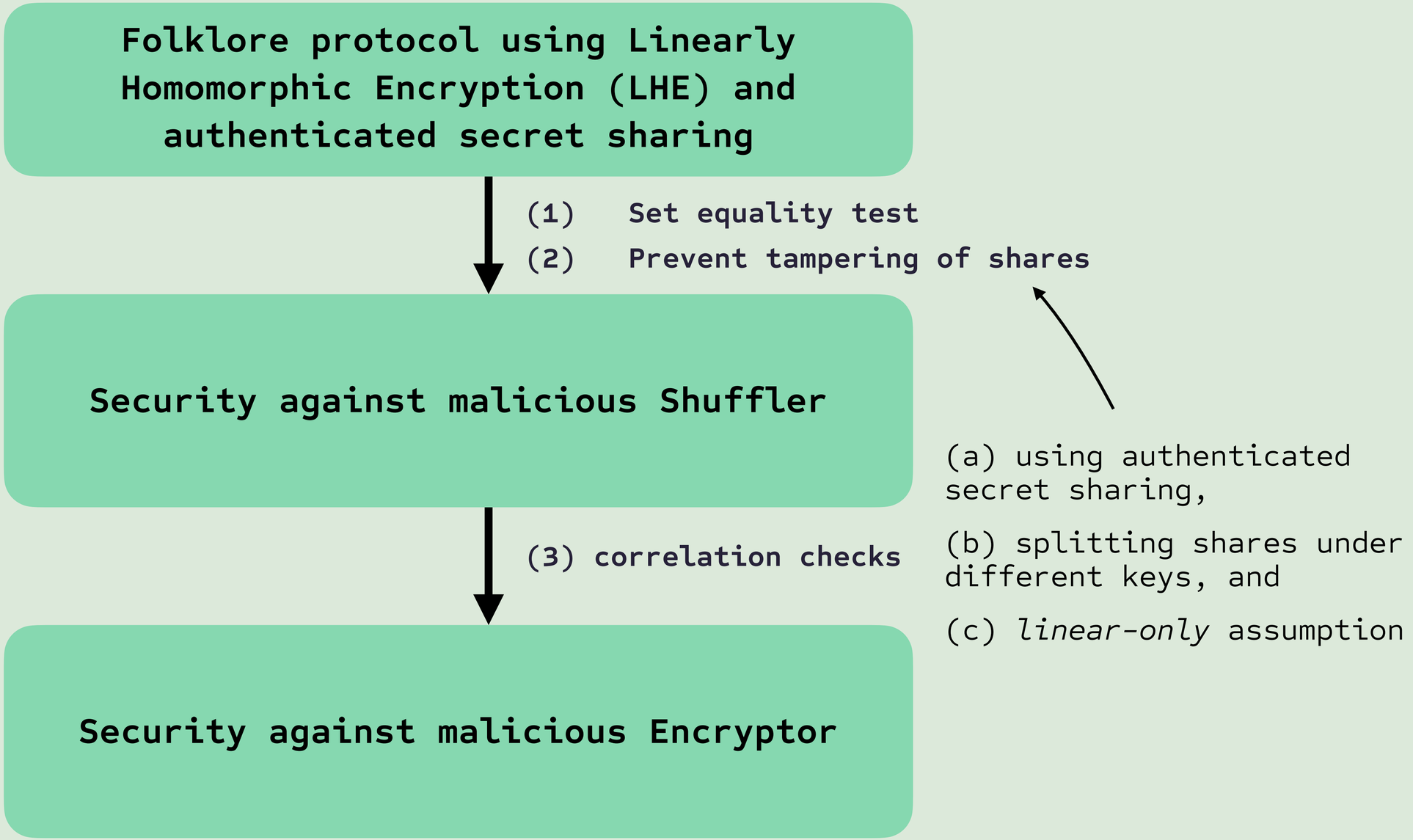
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Present a maliciously secure 3-party information-theoretic protocol also using authenticated secret sharing

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Communication-efficient variant

Also present a more efficient protocol in which shares can be delegated in unauthenticated way with additional checks

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Also present a more efficient protocol in which shares can be delegated in unauthenticated way with additional checks

Constant-round set equality test

Construct a constant-round set equality test using logarithmic derivative of the standard polynomial root test

(pending paper update)

Thank you!

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Linear Secret-shared Shuffle with Malicious Security (ePrint 2025/2137)